SHRIMATI INDIRA GANDHI COLLEGE

(Nationally Accredited at ‘A’ Grade (3rd Cycle) By NAAC)
Tiruchirappalli – 2.

QUESTION BANK FOR

M.Sc MATHEMATICS

2017-2018

DEPARTMENT OF MATHEMATICS

CONTENT

S.No. 2374 **P 16 MA 11**

(For candidates admitted from 2016-2017 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2016.

Mathematics

ALGEBRA

Time: Three hours **Maximum: 75 marks**

SECTION A – (10X2=20)

Answer ALL questions.

1. Define a normal subgroup with an example.

2. What is meant by an even permutation? Give a example.

3. If U is an ideal of R and 1 $\in$ U, then prove that U=R.

4. Define a Euclidean ring.

5. Show that x^2+1 is irreducible over the integers mod 7.

6. Define a unique factorization domain.

7. Define the following terms:

   (a) Algebraic over F

   (b) Algebraic of degree n.

8. If a $\in$ k is a root of p(x) $\in$ F[x], where F $\subset$ K, then prove that $(x-a) / p(x)$ in $k[x]$.

9. What is meant by the Galois group?

10. If the field F has p^m elements then prove that F is the splitting field of the polynomial $x^{p^m} - x$.

SECTION B – (5X5=25)

Answer ALL questions.

11. (a) Prove that HK is a subgroup of G if and only if HK=KH.

Or

(b) State and prove the Cauchy theorem for abelian groups.

12. (a) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R / M is a field.

Or

(b) If p is a prime number of the form 4n+1, then prove that $p = a^2 + b^2$ for some integers a,b.

13. (a) State and prove the division algorithm.

Or

(b) State and prove the Schwartz inequality.

14. (a) Prove that the polynomial of degree n over a field can have atmost n roots in any extension field.

Or

(b) If F is a field of characteristic $p \neq 0$, then prove that the polynomial $x^{p^m} - x \in F[x]$, for $n \geq 1$, has distinct roots.

15. (a) Show that the fixed field of G is a subfield of K.

Or

(b) Let K be a field and let G be a finite Subgroup of the multiplicative group of non zero elements of K. Prove that G is a cyclic group.

SECTION C – (3X10=30)

Answer any THREE questions.

16. State and prove the first sylow theorem. Also state the third sylow theorem.

17. (a) Let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.

(b) Show that J[i] is a Euclidean ring.

18. (a) State and prove the Eisenstein criterion theorem.

(b) Show that $W^{\perp}$ is a subspace of V.

19. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover, [L:F] = [L:K][K:F].

20. If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then find the elements a and b in F such that $1 + \alpha\alpha^2 + \beta\beta^2 = 0$.

M.Sc DEGREE EXAMINATION APRIL 2017

MATHEMATICS

GRAPH THEORY (P16MA14)

SECTION – A (10X2=20)

ANSWER ALL THE QUESTIONS.

1. Define a self-complementary graph with an example.
2. What is meant by tournament? Give an example for tournament on four vertices.
3. Define a cut vertex with an example.
4. Give an any two examples for isomorphic trees.
5. Define an independent number. Give an example.
6. What do you mean by Hamiltonian cycle? With suitable example.
7. Find the chromatic number of C_n.
8. What is meant by chromatic index of graph G ? Give an illustration.
9. Using Euler formula,show that K_5 is nonplanar.
10. State four color problem.

SECTION – B (5X5=25)

ANSWER THE FOLLOWING QUESTIONS.

11. (a) Prove that the number of edges of a simple graph with w components cannot exceed (n-w)(n-w+1)/2

OR

(b) Show that every tournament contains a directed Hamilton path.

12.(a) with the usual notation,prove that

$K(G) \leq \lambda(G) \leq \delta(G)$.

OR

(b) If e is not a loop of G,then prove that

$\tau(G)=\tau(G-e)+\tau(G.e)$.

13.(a) Determine the values of the parameters α,α^l,β and β^l for the Herschel graph.

OR

(b) Show that the closure Cl(G) of a graph G is well-defined.

14.(a) Define critical graph with an example.Also prove that every critical graph is a block.

OR

(b) Let G be any graph.Prove that

$f(G;\lambda)=f(G-e;\lambda)-f(G.e;\lambda)$ for any edge e of G.

15.(a) If the girth k of a connected plane graph G is atleast 3,then prove that

$m \leq K(n-2)/(K-2)$.

OR

(b) Show that a graph G is planar if and only if each of its blocks is planar.

SECTION – C (3X10=30)

ANSWER ANY THREE QUESTIONS.

16.Define the following terms with an illustrations.

(a) Union of two graphs;

(b) Intersection of two graphs;

(c) Join of two graphs;

(d) Cartesian product of two graphs;

(e) Kronecker product.

17.state and pove Cayley's formula.

18. Show that a graph is Eulerian if and only if each edge e of G belongs to an odd number of cycles of G.

19. State and prove the Vizing's theorem.

20.(a) Derive the Euler formula for plane graph.

(b) Show that every planar graph is 6-vertex colorable.

MSC DEGREE EXAMINATION NOVEMBER 2016

MATHEMATICS

GRAPH THEORY (P16MA14)

SECTION – A (10X2=20)

ANSWER ALL THE QUESTIONS.

1. Define a complete bipartite graph with an example.
2. Define the join of two graphs. Give an example.
3. Define a cut vertex. Give an example.
4. What is meant by a spanning tree? Give an example.
5. Define a matching of a graph. Also draw the Herschel graph.
6. Short notes on a Hamiltonian graphs.
7. What do you meant by critical graph? With suitable example.
8. Define a triangle-free graph. Give an example.
9. Show that the graph k_5 is non planar.
10. State the four-color theorem.

SECTION – B (5X5=25)

ANSWER THE FOLLOWING QUESTIONS.

11. ((a) If a simple graph G is not connected, then prove that G^c is connected.

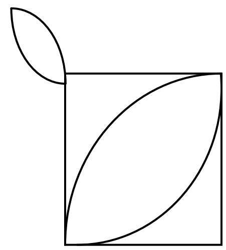
Or

(b) Show that every tournament of order n has at most one vertex v with $d^+(v)=n-1$.

12.(a) Show that an edge e=x y of a graph G is a cut edge of a connected graph G if and only if e does not belong to any cycle of G.

Or

(b) Find $\tau(G)$ for the following graph G:

G: 

13.(a) For any graph G for which $\delta>0$, Prove that $\alpha^{l} + \beta^{l} = n$.

Or

(b)If G is Hamiltonian, then prove that for every nonempty proper subset S of V, $\omega(G-S) \leq |S|$.

14.(a) If G is K-critical, then prove that $\delta \geq k-1$.

Or

(b) If G and H are disjoint, then prove that $f(G\cup H; \lambda)=f(G ; \lambda)\, f(H; \lambda)$.

15.(a) Show that a graph is planar if and only if it is embeddable on a sphere.

Or

(b) Prove that every planar graph is 6-vertex colorable.

SECTION – C (3X10=30)

ANSWER ANY THREE QUESTIONS.

16.(a) With the usual notations, prove that $\delta \leq 2m/n \leq \Delta$.

(b) Show that the number of edges of a simple graphs with w components cannot exceed $(n-w)(n-w+1)/2$.

17. State and prove the Cayley`s formula.

18. For a connected graph G. Prove that following statements are equivalent:

(a) G is Eulerian

(b) The degree of each vertex of G is an even positive integer.

(c) G is an edge-disjoint union of cycles.

19. With the usual notations, show that

$X^{1}(K_n)=\{$n-1 if n is even

n if n is odd

20. (a) State and prove the Euler formula for plane graph.

(b) If the girth k of a connected plane graph G is atleast 3, then prove that m $\leq$ k n-2)/(k-2).

S.NO.6426 **P 16 MA 15**

(For candidates admitted from 2016-2017 onwards)

M.Sc. DEGREE EXAMINATION, APRIL-2017

Mathematics

INTEGRAL EQUATIONS, CALCULUS OF

VARIATIONS AND TRANSFORMS

Time : Three hours Maximum : 75 Marks

PART A – (10×2=20)

Answer ALL questions.

1. Explain Lagrange's multipliers.
2. Write down the Euler equation.
3. Define Fourier cosine transform and its inversion Formula.
4. State Fourier integral theorem.
5. State Hankel Transform
6. Write down linearity property in Hankel Transform.
7. Define the Fredholm integral equation of the I kind
8. Define Degenerate Kernel.
9. Find the Neumann series for the solution of the Integral equation
10. State Fredholm's first theorem.

PART B- (5×5=25)

Answer ALL questions.

11. (a) Find the minimal surface of revolution passing through two given points, minimize the integral

$\frac{1}{2\pi} = \int_{x_1}^{x_2} y(1 + y'^2)^{\frac{1}{2}} \; dx$. (OR)

b) Determine the curve of length L which passes through the points (0,0) and (1,0) and for which the area I between the curve and the x-axis is a maximum.

Here $L = \int_2^1 (1 + y'^2)^{\frac{1}{2}} \ dx.$

12. (a) Find the finite Fourier sine and cosine transform of the function

f(*x*) = *x*. *(OR)*

(b) Find the finite Fourier cosine transform of $f(x)$ if $f(x) = -\frac{cos(\pi - x)}{k \ sink\pi}$.

13. (a) Find the Hankel transform of

$$f(x) = \begin{cases} 1, & 0 < x < a, n = 0 \\ 0, & x > a, n = 0 \end{cases}$$

(OR)

(b) Find Hankel transform of $\frac{1}{r}$ and then apply the inversion formula to get the original function.

14a)Explain the scalar product of two functions. (OR)

b)Solve the Fredholm integral equation of the II kind g(s) =f(s) + $\lambda \int_0^1 (st^2 + s^2t) g(t) dt$.

15.(a) Solve the integral equation

$$g(s) = f(s) + \lambda \int_0^1 e^{s-t} \, g(t) \, dt. \qquad \text{(Or)}$$

(b) Show that $D(s, t{:}\,\lambda)$ satisfies, for real values of the integral differential equation.

$$D(\lambda) \frac{\partial}{\partial \lambda} D(s, t; \lambda) = D'(\lambda) D\,(s, t; \lambda) \ + \int D(s, x; \lambda) D(x, t; \lambda) dx.$$

PART C- (3×10=30)

Answer any THREE questions.

16. Derive Rayleigh's principle.

17.Find the finite sine transform of *f(x)* if (a) $f(x) = \cos kx$

(b) $f(x)=x^2$

18. Derive the Hankel transform of

$$f(x) = \begin{cases} a^2 - x^2, \theta < x < a, n = 0 \\ 0 \qquad , x > a , n = 0 \end{cases}$$

19. Find the eigen values and eigen functions of the homogenous integral equation $g(s) = \lambda \int_1^2 \left(st + \frac{1}{st}\right)\left(g(t)\right) dt.$

20. Solve the Fredholm integral equation $g(s) = 1 + \lambda \int_0^1 (1 - 3st) g(t) dt$. Evaluate the resolvent Kernel.

S. No 6424 **P16MA13**

(For Candidates admitted from 2016-2017 onwards)
M.Sc DEGREE EXAMINATIONS, APRIL 2017
Mathematics
ORDINARY DIFFERENTIAL EQUATIONS

Time: Three hours **Maximum:75 marks**

PART A - (10×2=20)

Answer ALL questions

1. Show that $y = c_1e^{2x} + c_2xe^{2x}$ is the general solution of $y'' - 4y' + 4y = 0$ on any interval.
2. Define ordinary point.
3. Define n[th] Legendre polynomial $P_n(x)$.
4. Write the Bessel function of the first kind of order 1.
5. State the Picard's theorem.
6. Give an example of non homogeneous linear system of differential equation.
7. State Sturm comparison theorem.
8. Define eigen functions.
9. Define critical point.
10. What is autonomous system.

PART-B (5×5=25)

ANSWER ALL QUESTIONS

11. a) If $y_1(x)$ and $y_2(x)$ are any two solutions of equation $y'' + p(x)y' + 2(x)y = 0$ on $[a, b]$ prove that their
Wronskian $w = w(y_1, y_2)$ is either identically zero or never zero on $[a, b]$.

Or

(b) Locate and classify the singular points on the x-axis of differential equation $(3x + 1)xy" - (x + 1)y' + 2y = 0$.

12. (a) Prove that $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\, cosx$.

Or

(b) If $p(x)$ is a polynomial of degree $n \geq 1$ s.t. $\int_{-1}^{1} x^k p(x)dx = 0$ for $k = 0,1,\dots,n-1$ show that $p(x) = CP_n x$ for some constant C.

13. (a) Show that $x = e^{4t}$, $y = e^{4t}$ and $x = e^{-2t}$ $y = -e^{-2t}$ are solutions of the homogeneous System $\frac{dx}{dt} = x + 3y, \frac{dy}{dt} = 3x + y$.

(Or)

(b) Find the exact solution of the initial value problem $y' = 2x(1 + y)$, $y(0) = 0$.

Starting with $y_0(x) = 0$, calculate $y_1(x), y_2(x)$ $y_3(x), y_4(x)$ and compare these results with the exact solution.

14. (a) Let $y(x)$ be a non trivial solution of equation $y'' + q(x)y = 0$ on a closed interval $[a, b]$. Prove that $y(x)$ has at most a finite number of zeros in this interval.

(Or)

(b) Find the eigen values λ_n and eigen functions $y_n(x)$ for the equation $y'' + \lambda y = 0$ for the case $y(0) = 0, y(2\pi) = 0$.

15. (a) Determine the nature and stability properties of the critical point (0,0) for

$\frac{dx}{dt} = 5x + 2y, \frac{dy}{dt} = -17x - 5y$

(Or)

(b) Show that the function $E(x, y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a > 0$ and $b^2 - 4ac < 0$, positive definite if and only if $a > 0$ and $b^2 - 4ac < 0$, and is negative definite, if an only if $a < 0$ and $b^2 - 4ac < 0$

PART C – (3x10=30)

Answer any THREE questions

16. Find the general solution of the equation

$(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2$

17. Show that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ & \end{cases}$

$\frac{2}{2n+1} \quad if\ m = n$

18. Let $f(x,y)$ be a continuous function that satisfies a Lipschitz Condition

$|f(x_1y_1) - f(x_2y_2)| \leq k|y_1 - y_2|$ on a strip defined by $a \leq x \leq b$ and $-\infty < y < \infty$. If $x_0, y_0)$ is any point of the strip, prove that the initial value problem $y' = f(x,y),\ y(x_0) = y_0$ has one and only solution $y = y(x)$ on the interval $a \leq x \leq b$.

19. Derive one dimensional wave equation.

20. Consider the system $\frac{dx}{dt} = a_1x + b_1y$, $\frac{dy}{dt} = a_2x + b_2y$. If $m_1\ and\ m_2$ are the roots of the auxiliary equation of the system , $m_1\ and\ m_2$ real, distinct and of the same sign, prove that the critical point (0,0) is a node.

S. No 2376 **P16MA13**

(For Candidates admitted from 2016-2017 onwards)

M.Sc DEGREE EXAMINATIONS NOVEMBER 2016

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

Time: Three hours **Maximum:75 marks**

PART A - (10×2=20)

Answer ALL questions

1. Show that $y = c_1e^{2x} + c_2xe^{2x}$ is the general solution of $y'' - 4y' + 4y = 0$ on any interval.
2. Define Airy functions.
3. Determine the nature of the point x=0 for the equation
4. $x^4y'' + (sinx)y = 0$
5. Write the Bessel's equation of order $\frac{1}{2}$.
6. Show that $f(x, y) = xy^2$ satisfies a Lipschitz condition on any rectangle $a \leq x \leq b \; and \; c \leq y \leq d$.
7. Write down the statement of Picard's theorem.
8. State the Sturm comparison theorem.
9. Find the Eigen values λ_n and Eigen functions $y_n(x)$ for the equation $y'' + \lambda y = 0, y(a) = 0, y(b) = 0$ when $a < b$.
10. Define spiral.
11. What is autonomous system.

PART-B (5×5=25)

ANSWER ALL QUESTIONS

11. (a) Verify that $y_1 = x^2$ is one solution of $x^2y'' + xy' - 4y = 0$ and find y_2 and the general solution.

(OR)

(b) Find the series solution of the equation $y'' + y = 0$.

12. (a) Find the general solution of the equation $(x^2-1)y'' + (5x+4)y' + 4y = 0$ near $x = -1$.

(OR)

(b) Show that

(i) $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$

(ii) $\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$

13. (a) Find the general solution of the system

$$\frac{dx}{dt} = x + y\,, \frac{dy}{dt} = 4x - 2y$$

(OR)

(b) Find the exact solution of the initial value problem $y' = x + y\,, y(0) = 1$. Starting with $y_0(x) = 1$, calculate $y_1(x), y_2(x), y_3(x)$.

14. (a) If $q(x) < 0$ and if $u(x)$ is a non trivial solution of $u'' + q(x)u = 0$, then prove that $u(x)$ has at most one zero.

(OR)

(b) Show that $y(x,t) = F(x+at) + G(x-at)$

satisfies the wave Equation $a^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$.

15. (a) Prove that (0,0) is stable critical point for the system

$$\frac{dx}{dt} = -2xy, \frac{dy}{dt} = x^2 - y^{3}.$$

(OR)

(b) Show that (0,0) is an asymptotically stable critical

point of $\frac{dx}{dt} = -y - x^3, \frac{dy}{dt} = x - y^3$.

PART C – (3x10=30)

Answer any THREE questions

16. Find two linearly independent series solutions of Chebyshev's equation

$(1-x^2)y'' - xy' + p^2y = 0$ for $|x| < 1$, where p is a constant.

17. Prove that $\int_{-1}^{1} P_m(x)P_n(x)\,dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$

18. Show that Wronskian of the two solution

$x = e^{at}(A_1cosbt - A_2sinbt)$

$y = e^{at}(B_1cosbt - B_2sinbt)$ and

$x = e^{at}(A_1sinbt + A_2cosbt)$

$y = e^{at}(B_1sinbt + B_2cosbt)$

is given by $W(t) = (A_1B_2 - A_2B_1)e^{2at}$ and prove that

$A_1B_2 - A_2B_1 \neq 0.$

19. Derive one dimensional wave equation.

20. If there exists a Liapunov'S function $E(x,y)$ for the system $\frac{dx}{dt} = F(x,y)$ $\frac{dy}{dt} = G(x,y)$, then prove that the Critical point (0,0) is stable. Furthermore, if this function has the additional property that the function $\frac{\partial E}{\partial x}F + \frac{\partial E}{\partial y}G$ is negative definite, then the critical point (0,0) is asymptotically stable.

S. No 2375 **P16MA12**

(For Candidates admitted from 2016-2017 onwards)

M.Sc DEGREE EXAMINATIONS NOVEMBER 2016

Mathematics

REAL ANALYSIS

Time: Three hours **Maximum:75 marks**

PART A - (10×2=20)

Answer ALL questions

1. Define the cantor set. Give an example.
2. Define a power series. Compute the radius of convergence of the power series
3. What is the difference between continuity and uniform continuity?
4. Let f be defined on [a, b] If f is differentiable at a point $x \in [a, b]$,then prove that f is continuous at x.
5. If p* is a refinement of p, then prove that U(p*,f,α)≤U(P,f,α)
6. If f maps [a,b] in to R^K and if $f \in R(\alpha)$ for some mototonically increasing function α on [a,b] then prove that $|f| \in \Re(\alpha)$.
7 .Define Equicontinuous families of functions with an example.
8. State Arzele- Ascoli theorem.
9. What is meant by linear transformation?
10.State the implicit function theorem.

PART-B (5×5=25)

ANSWER ALL QUESTIONS

11. (a) Show that $d(x, y) = \frac{|x, y|}{1 + |x - y|}$ defines a metric on the real line

(or)

(b)State and prove ratio test for the convergence of a series.

12. (a) If f is continuous mapping of a metric space X into a metric space Y, and if E is a connected subset of X, prove that $f(E)$ is connected.

(or)

(b) State and prove Cauchy mean value theorem.

13. (a) If $f_1, f_2 \in \mathscr{R}(\alpha)$ on [a,b] then prove the following:

(i) $f_1 + f_2 \in \mathscr{R}(\alpha)$;

(ii) $cf \in \mathscr{R}(\alpha)$; (iii) $\int_a^b (f_1 + f_2)d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$.

(or)

(b) State and prove the fundamental theorem of calculus.

14. (a) Let $f_n(x) = \frac{sinnx}{\sqrt{n}}$ (x real, n=1,2,……). Prove that

$\{f_n'\}$ does not converge to f' where

$f(x) = \lim_{n\to\infty} f_n(x)$

(or)

(b) Show that there exists a real continuous on the real line which is nowhere differentiable.

15. (a) Let Ω be the set of all invertible linear operators on R^n.

If $A \in \Omega, B \in L(R^n)$ and $\|B - A\|.\|A^{-1}\| < 1$, then prove that

$B \in \Omega$.

(or)

(b)If X is a complete metric space, and if ϕ is a

contraction of X into X, then prove that there exists

one and only one $x \in X$ such that $\phi(x) = x$.

PART C – (3x10=30)

Answer any THREE questions

16. Define and prove that it is not rational . Also prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$.

17. State and prove L'hospital's rule .

18. (a) Define Riemann-Stieltjes integral of f with respect to α over $[a,b]$. When does it coincide with Riemann integral of f over $[a,b]$.

(b) If f is continuous on $[a,b]$ then prove that $f \in \Re(\alpha)$ on $[a,b]$.

19. State and prove the Stone-Weierstrass theorem .

20. State and prove the inverse function theorem .

S.No.6427 **P16MA21**

(For candidates admitted from2016-2017 onwards)

M.Sc. DEGREE ,EXAMINATION, APRIL 2017

Mathematics –Major

COMPLEX ANALYSIS

Time:Three hours **Maximum:75 marks**

SECTION A-(10x2=20)

Answer ALL Questions

1. When will you say a set is totally bounded?
2. Distinguish between translation, rotation and inversion.
3. Compute: $\int_{|z|=1} e^z z^{-1} dz$
4. State Liouville's theorem.
5. Define: Zero and pole.
6. State the maximum principle theorem.
7. Obtain the residue for $\dfrac{e^z}{(z-a)(z-b)}$
8. State the Rouche's theorem.
9. Prove that the arithmetic mean of a harmonic functions over concentric circles $|z| = r$ is a linear function of log r.
10. State the Laurent series.

SECTION B –(5x5=25)

Answer ALL Questions

11. (a) Prove : on a compact set every continuous function is uniformly Continuous.

Or

(b) If $T_1(z) = \dfrac{z+2}{z+3}, T_2(z) = \dfrac{z}{z+1}$,find $T_1, T_2(z)$, $T_2, T_1(z)$ and $T_1^{-1}, T_2(z)$.

12. (a) With the usual notations ,prove that $\left|\int_a^b f(t)dt\right| \le \int_a^b |f(t)|dt$.

Or

(b) State and prove cauchy's integral formula.

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

(b) State and prove the Schwarz lemma.

14. (a) State and prove the residue theorem.

Or

(b) When will you say a set is homologous to zero?Also state and prove the Argument principle.

15. (a) If u_1 and u_2 are harmonic in a region Ω,then prove that

$\int_\gamma u_1^* du_2 - u_2^* du_1 = 0$ for every cycle γ which is homologous to zero in Ω.

Or

(b) State and prove the Taylor series.

SECTION C – (3x10=30)

Answer any THREE questions.

16. (a) Is an analytic function f(z) with a constant modulus reduces to a constant? If yes, Justify your answer.

(b) Define cross ratio.If z_1, z_2, z_3, z_4 are distinct points in the extended plane and T be any linear transformation then prove that $(T_{z_1}, T_{z_2}, T_{z_3}, T_{z_4}) = (z_1, z_2, z_3, z_4)$.

17. State and prove the Cauchy's theorem for a rectangle.

18. State and Prove the Local mapping theorem.

19. Show that $\int_0^\pi \log \sin x dx = -\pi \log 2$.

20. Establish the Poisson formula $u(a) = \frac{1}{2\pi} \int_{|z|=r} \frac{R^2 - |a|^2}{|z-a|^2} u(z) d\theta$.

S.N0.6432 P 16 MAE 1C

(For candidates admitted from 2016-2017 onwards)
M.Sc DEGRE EXAMINATON APRIL 2017
MATHEMATICS – Elective (P16 MAE 1 C)

FUZZY SETS AND THEIR APPLICATIONS

Part – A(10x2=20)
Answer All questions

1. Differentiate between a crisp set and fuzzy set
2. Define a strong α cut with an example
3. Write down the standard fuzzy operations
4. When will you a point 'a' is dual point
5. Find the value of
   (a) [2,5] - [1,3]
   (b) [4,10] / [1,2]
6. If $A=[a_1,a_2]$, $B=[b_1,b_2]$ 0=[0,0] and 1=[1,1] then prove that A+B = B+A and A.B = B.A
7. Write down the algorithm for transitive closure $R_r(X,X)$
8. Define a fuzzy partial ordering
9. What is meant by multiperson decision making
10. Write down the mathematical formulation of fuzzy linear programming

Part- B(5x5=25)
Answer All questions

11. a) Find the scalar cardinality and fuzzy cardinality for the following fuzzy set
    $c(x) = \frac{x}{x+1}$ for $x \in \{0,1,2,3,4,5\ldots\ldots.10\}=X$
    (OR)

    b) Let $f:X \rightarrow Y$ be an arbitrary crisp functions and let $A_i \in F(X)$ and any $B_i \in F(Y)$, $i \in I$. Prove that (i) $f(\cup_{i\in I} A_i) = \cup_{i\in I} f(A_i)$
    (ii) $f(\cup_{i\in I} A_i) \leq \cap_{i\in I} f(Ai)$
12. a) Show that every fuzzy complement has at most one equilibrium
    (OR)
    b) Let <i,u,c> be a dual triple that satisfies the law of excluded middle and the law of contradiction. Prove that <i,u,c> does not satisfy the distributive laws
13. a) Determine which fuzzy sets defined by the following functions are fuzzy functions
    (i) $A(x) = \begin{cases} \sin(x) & \text{for } 0 \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases}$

(ii) $B(x) = \begin{cases} x & \text{for } 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$

(iii) $D(x) = \begin{cases} \min(1, x) & \text{for } x \ge 0 \\ 0 & \text{for } x < 0 \end{cases}$

(iv) $E(x) = \begin{cases} 1 & for\ x = 5 \\ 0 & \text{otherwise} \end{cases}$

(OR)

(b) Explain the arithmetic operations on fuzzy numbers with suitable example

14.(a) What is meant by a sagittal diagram? Explain with suitable illustration

(OR)

(b) Determine the complete α – covers of the compatibility relation whose membership matrix is given below

$$\begin{bmatrix} 1 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0.8 & 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0.8 & 1 & 0.7 & 0.5 & 0.7 & 0 \\ 0 & 0 & 0 & 0.7 & 0.7 & 1 & 0.4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

15.(a) Enumerate the individual decision making with an illustration

(OR)

(b) What is meant by Haming distance of fuzzy numbers? Explain with an example

Part- C (3x10=30)

Answer Any THREE questions

16. Consider the fuzzy sets A,B and C defined on the interval X=[0,10] of real numbers by themembership grade functions

$A(x) = x/(x+10)$

$B(x) = 2^{-x}$ $C(x) = 1/(1+10(x-2)^2)$. Determine the following

(a) $\bar{A}$,$\bar{B}$,$\bar{C}$ (b) A∪B∪C (c) A∩B∩C (d) A∩ $\bar{C}$,$\overline{B \cap C}$

17. Let u_w denote the class of Yager t- conforms defined by $u_w(a,b) = \min(1, (a^w+b^w)^{1/w})$, (w>0).Prove that $\max(a,b) \le u_w(a,b) \le u_{max}(a,b)$ for all $a,b \in [0,1]$

18. Let A⊆ E and B⊆ F.Prove that following

(a) A+B ⊆ E+F

(b) A-B⊆E-F

(c) A.B⊆E.F

(d) A/B⊆E/F where A,B,E and F are closed intervals

19. Enumerate the following types of fuzzy relations with suitable example for each

(a) Reflexive

(b) Symmetric

(c) Transitive

(d) Anti symmetric

(e) Anti reflexive

20. Solve the following fuzzy linear programing problem:

Maximum $z = 6x_1+5x_2$

Subject to

$<5,3,2>x_1+<6,4,2>x_2 \leq <25,6,9>$

$<5,2,3>x_1+<2,1.5,1>x_2 \leq <13,7,4>$

$x_1, x_2 \geq 0$

M.Sc Degree Examination

April 2017

MATHEMATICS

Linear Algebra P16MA22

PART-A (10X2=20)

ANSWER ALL QUESTIONS

1. Write down three elementary operations m*n matrix A over the field
2. Define a row-echelon matrix
3. If $\alpha_1,\ldots\ldots\alpha_n$ are vectors in V and $c_1,\ldots\ldots c_n$ are scalars, then prove that
    a. $T(0) = 0$
    b. $T(c_1\alpha_1+\ldots\ldots\ldots+c_n\alpha_n) = c_1T(\alpha_1)+\ldots\ldots+ c_nT(\alpha_n)$
4. Let F be a field and Let T be the operator on F^2 defined by $T(x_1,x_2)=(x_1,0)$. Prove that T is linear operator on F^2
5. Define linear algebra over the field
6. Let F be a subfield of complex numbers and let A be the following 2*2 matrix $A=\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$. Compute f(A) if $f=x^2-x+2$
7. Let K be the ring of integers and $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Then prove that A is not invertible as a matrix over the ring of integers.
8. Prove that similar matrices have the same characteristic polynomial
9. Suppose that E is a projection. Let R be the range of E and N be the null space of E. Prove that the vector β is the range R iff $E\beta= \beta$
10. Give an example for an invariant under transformation

PART-B (5X5=25)

ANSWER ALL QUESTIONS

11.a) If A and B are row-equivalent m*n matrices, then prove that the homogeneous systems of linear equations AX=0 and BX=0 have exactly the same solutions

b) If A is an n*n matrix, then prove that A is row equivalent to the n*n identity matrix iff the system of equations AX=0 has only the trivial solutions

12.a) Let V and W be vectors spaces over the field F and let T be a linear transformation from V to W. Suppose that V is finite dimensional. Then prove that rank (T) + nullity (T) = dim V

b) Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n

13.a) If F is a field containing on infinite number of distinct elements, then prove that the mapping $f \rightarrow \tilde{f}$ is an isomorphism of the algebra of polynomials over F on to the algebra of polynomial functions over F

b) (i) Define n-linear function

(ii) Prove that a linear combination of n-linear function is n-linear

14.a) Let K be a commutative ring with identity and let A and B be n*n matrices over F. Then prove that det(AB) =(detA)(detB)

b) Let T be a linear operation on an n-dimensional vector space V. Prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities

15.a) Let V be a finite dimensional vector space. Let $W_1,......W_k$ be subspaces of V and let $W=W_1+.......+W_k$. Prove that the following α equivalent

(i) $W_1,......W_k$ are independent

(ii) For each j,$2\leq j\leq k$ we have $W_j\cap(W_1+.........+W_{j-1}) = \{0\}$

b) Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is triangulable iff the minimal polynomial for T is a product of linear polynomials over F

PART-C (3X10=30)

ANSWER ANYTHREEQUESTIONS

16. If W_1 and W_2 are finite dimensional subspaces of a vector space V, prove that $W_1+ W_2$ is finite dimensional and dim W_1 +dim W_2 = dim $(W_1\cap W_2)$+dim $(W_1+ W_2)$

17. Let V be a n-dimensional vector space over the field F and let W be an m-dimensional vector space over F. Then prove that the space L(V,W) is finite dimensional and had dimension mn

18. If F is a field then prove that a non scalar monic polynomial in F(x). Can be factored as a product of monic primes in F(x) in one and except for order only one way

19. State and prove Cayley-Hamilton Theorem

20. State and prove Primary decomposition theorem

M.SC. DEGREE EXAMINATION, APRIL 2017
Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

PART A-(10*2=20)

Answer ALL questions.

1.From a partial differential by eliminating the arbitrary constants `a' and 'b' from $2z=(ax+y)^2+b$.

2. Verify that the equation $z=\sqrt{2x+a}+\sqrt{2y+b}$ is a complete integral of the partial differential equation $z=1/p+1/q$.

3. Prove that along every characteristic strip of the equation $F(x,y,z,p,q)=0$, the function $F(x,y,z,p,q)$ is a constant.

4. Find the complete integral of the equation $(p+q)(z-xp-yq)=1$.

5. Verify that the partial differential equation $\partial^2z/\partial x^2-\partial^2z/\partial y^2=2z/x$ is satisfied by $z=1/x\phi(y-x)+\phi^1(y-x)$ where ϕ is an arbitrary function.

6. write down the poisson equation in Cartesian form.

7. classify the partial differential equation $u_{xx}+u_{yy}=u_{zz}$.

8. Define uniform non linear equation.

9. Define exterior Neumann problem.

10. Define interior dirichlet problem.

PART B-(5*5=25)

Answer ALL questions.

11 (a). Form a partial differential equation by eliminating the arbitrary function f from $z=f(xy/z)$

(Or)

(b). Find the general solution of the differential equation $x^2\partial z/\partial x+y^2\partial z/\partial y=(x+y)z$.

12 (a). Find the complete integral of the equation $p^2x+q^2y=z$.

(Or)

(b). Find the complete integral of the partial differential equation $(p^2+q^2)x=pz$ and deduce the solution which passes through the curve $x=0$, $z^2=4y$.

13 (a). Find the particular integral of the equation $(D^2-D^l)z=2y-x^2$.

(Or)

(b). If $\beta_r D^l+\gamma_r$ is a factor of $F(D,D^l)$ and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ, then if $\beta_r \neq 0$, prove that $u_r=\exp(-\gamma_r y/\beta_r)\phi_r(\beta_r x)$ is a solution of the equation $F(D,D^l)=0$.

14 (a). solve the equation $z(qs-pt)=pq^2$.

(Or)

(b). solve $\partial^2 z/\partial x^2+\partial^2 z/\partial y^2=1/k\ \partial z/\partial t$.

15 (a). prove that $r\cos\theta$ and $r^{-2}\cos\theta$ satisfy Laplace's equation, when r,θ,ϕ are spherical polar coordinates.

(Or)

(b). show that the surfaces $x^2+y^2+z^2=cx^{2/3}$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.

PART C-(3*10=30)

Answer ALL questions.

16. Find the surface which intersects the surface of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1$, $z=1$.

17. show that the equations $xp=yq$, $z(xp+yq)=2xy$ are compatible and solve them.

18. Reduce the equation

$$Y^2\ \partial^2 z/\partial x^2-2xy\ \partial^2 z/\partial x\partial y+x^2\ \partial^2 z/\partial y^2=y^2/x\ \partial z/\partial x+x^2/y\ \partial z/\partial y$$

to canonical form and hence solve it.

19. Derive the solution of the equation $\partial^2 V/\partial r^2+1/r\ \partial V/\partial r+\partial^2 V/\partial z^2=0$ for the region $r\geq 0, z\geq 0$ satisfying the conditions.

(a) $V\to 0$ as $z\to\infty$ and as $r\to\infty$

(b) $V=f(r)$ as $z=0$, $r\geq 0$.

20. Find the potential function $\psi(x,y,z)$ in the region $0\leq x\leq a$, $0\leq y\leq b$, $0\leq z\leq c$ satisfying the conditions.

(a) $\boldsymbol{\psi}=0$ as $x=0, x=a, y=0, y=b, z=0$

(b) $\boldsymbol{\psi}=f(x,y)$ as $z=c, 0\leq x\leq a, 0\leq y\leq b$.

M. Sc DEGREE EXAMINATION

STOCHASTIC PROCESSES-April 2017

SUB.CODE: P16MAE2A **MAX.MARKS:75**

PART- A (2X10=20)

ANSWER ALL THE QUESTIONS

1. Define Markov chain.
2. Define Homogeneous.
3. Define Periodicity.
4. Define Closed State.
5. Write down the mean and variance of Poisson process.
6. State the additive property of Poisson process.
7. Define Renewal period.
8. Define renewal function.
9. Write down the Little's formula.
10. State rate equality principle.

PART -B (5X5=25)

ANSWER THE FOLLOWING QUESTIONS.

11. a) Let $\bar{X}n, n \geq 0$ be a Markov chain with three states 0,1,2 with transition matrix $\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$

and the intial distribution $p_r\left\{X_0 = i\right\} = \frac{1}{3}, i = 0,1,2.$ Find $p_r\{X_3=1,$ $X_2 = 1, X_1 = 1, X_0 = 2\}$.

(OR)

b) Suppose that the probability of dry day (state 0) following a rainy day (state1) is 1/3 and the probability of rainy day following dry day is 1/2 .Given that May 1 is dry day Find the probability of May 3 and May 5 (Both are dry day).

12. a) State and prove first entrance theorem

(OR)

b) Classify the Markov Chain whose transition matrix is

1 2 3 4

$$P = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{12} \end{pmatrix}$$

13. a) Explain the Postulates for Poisson Process.

(OR)

b) State and Prove the estimation of the parameter of Poisson process.

14.a)Prove the following theorem:The distribution of N(t) is given by $P_n(t) = P_r\{N(t) = n\} = F_n(t) - F_{n+1}(t)$ and the expected number of arrivals by $M(t) = \sum_{n=1}^{\infty} F_n(t)$.

(OR)

b) State and Prove Wald's equation.

15. a) Explain waiting time process.

(OR)

b) Explain Sojourn time.

PART - C (3X10=30)

ANSWER ANY THREE QUESTIONS.

16) Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_{n,} n \geq 1$ be the digit leaving the nth stage of system and $, X_0$ be the digit entering the first stage (leaving the 0th stage).At each stage there is a constant probability q that the digit enters will be transmitted unchanged (ie., the digit changes when it leaves),p+q=1.Find the probability that the digit entering the first stage 0 given that the digit leaving the mth stage is 0.

17) Classify the Markov chain where t.p.m is

$$\begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ p = \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

18. State and prove the decomposition of a Poisson process.
19. State and prove Elementary Renewal theorem.
20. Describe Waiting time in the queue.

******* ALL THE BEST *******