Generation Formula for Integer Solutions to Special Elliptic Paraboloids

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Abstract

Knowing a solution of ternary quadratic diophantine equation representing elliptic paraboloid, a general formula for generating sequence of solutions based on the given solution is illustrated.

Keywords

ternary quadratic, generation of solutions, elliptic paraboloid.

2010 Mathematics Subject Classification: 11D09

I. INTRODUCTION

The subject of diophantine equations in number theory has attracted many mathematicians since antiquity. It is well-known that a diophantine equation is a polynomial equation in two or more unknowns with integer coefficients for which integer solutions are required. An integer solution is a solution such that all the unknowns in the equation take integer values. An extension of ordinary integers into complex numbers is the gaussian integers. A gaussian integer is a complex number whose real and imaginary parts are both integers. It is quite obvious that diophantine equations are rich in variety and there are methods available to obtain solutions either in real integers or in gaussian integers.

A natural question that arises now is, whether a general formula for generating sequence of solutions based on the given solution can be obtained? In this context, one may refer [1-7]. The main thrust of this communication is to show that the answer to the above question is in the affirmative in the case of the following ternary quadratic diophantine equations, each representing a elliptic paraboloid.

II. METHOD OF ANALYSIS

Illustration: I

The ternary quadratic diophantine equation under consideration is

$$16x^2+9y^2=4z \tag{1}$$

Let (x_0, y_0, z_0) be any solution of (1).

The solution may be in real integers or in gausssian integers or in irrational numbers.

Let (x_1, y_1, z_1) be the second solution of (1), where

$$x_1=h_0-x_0\ ,\ y_1=h_0-y_0\ ,\ z_1=z_0+6h_0^2 \tag{2}$$

in which h_0 is an unknown to be determined.

Substitution of (2) in (1) gives

$$h_0=32x_0+18y_0 \tag{3}$$

Using (3) in (2), the second solution is given by

$$x_1=31x_0+18y_0\ ,\ y_1=32x_0+17y_0 \tag{4}$$

$$z_1=z_0+6(32x_0+18y_0)^2 \tag{5}$$

Let (x_2, y_2, z_2) be the third solution of (1), where

$$x_2=h_1-x_1\ ,\ y_2=h_1-y_1\ ,\ z_2=z_1+6h_1^2$$

in which h_1 is an unknown to be determined.

The repetition of the above process leads to

$$h_1=7^2h_0\ ,\ x_2=1537x_0+864y_0\ ,\ y_2=1536x_0+865y_0 \tag{6}$$

$$z_2=z_0+6(32x_0+18y_0)^2\left(1+49^2\right) \tag{7}$$

Observations on Two Special Hyperbolic Paraboloids

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Abstract: Knowing a solution of ternary quadratic diophantine equation representing hyperbolic paraboloid, a general formula for generating sequence of solutions based on the given solution is illustrated.

KEYWORDS: Ternary quadratic, generation of solutions, hyperbolic paraboloid

I. INTRODUCTION

The subject of diophantine equations in number theory has attracted many mathematicians since antiquity. It is well-known that a diophantine equation is a polynomial equation in two or more unknowns with integer coefficients for which integer solutions are required. An integer solution is a solution such that all the unknowns in the equation take integer values. An extension of ordinary integers into complex numbers is the gaussian integers. A gaussian integer is a complex number whose real and imaginary parts are both integers. It is quite obvious that diophantine equations are rich in variety and there are methods available to obtain solutions either in real integers or in gaussian integers.

A natural question that arises now is, whether a general formula for generating sequence of solutions based on the given solution can be obtained? In this context, one may refer [1-7]. The main thrust of this communication is to show that the answer to the above question is in the case of the following ternary quadratic diophantine equations, each representing a hyperbolic paraboloid.

II. METHOD OF ANALYSIS

Hyperbolic Paraboloid: 1

Consider the hyperbolic paraboloid given by

$$(a+1)x^2 - ay^2 = 2z \tag{1}$$

Introduction of the linear transformations

$$x = X \pm aT \ , \ y = X \pm (a+1)T \tag{2}$$

leads to

$$X^2 = (a^2+a)T^2 + 2z$$

which is satisfied by

$$T = 4k \ , \ z = 2k^2 \Rightarrow X = 2k(2a+1)$$

In view of (2), we have

$$x = 8ka + 2k \ ,2k \ \ and \ \ y = 8ka + 6k \ , -2k \tag{3}$$

Denote the above values of x, y, z as x_0, y_0, z_0 respectively. We illustrate a process of obtaining sequence of integer solutions to the given equation based on its given solution (3).

Let (x_1, y_1, z_1) be the second solution of (1), where

$$x_1 = h - x_0 \ , \ y_1 = y_0 + h \ , \ z_1 = z_0 + h \tag{4}$$

in which h is an unknown to be determined.

Substitution of (4) in (1) gives

$$h = 2(a+1)x_0 + 2ay_0 + 2 \tag{5}$$

Using (5) in (4), the second solution (x_1, y_1, z_1) of (1) is expressed in the matrix form as

$$(x_1, y_1, z_1)^t = M(x_0, y_0, z_0)^t$$

where t is the transpose and

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On Heron Triangles

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Abstract: *Different set of formulas for integer heron triangles are obtained.*
Keywords - *Heron triangles, Heron triples, Isosceles heron triangles.*

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I. Introduction

The numbers that can be represented by a regular geometric arrangement of equally spaced points are called the polygonal numbers or Figurate numbers. Mathematicians from the days of ancient Greeks have always been interested in the properties of numbers that can be arranged as a triangle, which is a three-sided polygon. There are many different kinds of triangles of which heron triangle is one. A heron triangle is a triangle having rational side lengths and rational area [1]. One may refer [2, 3] for integer heron triangles. If a, b, c are the sides of the heron triangle then the triple (a, b, c) is known as Heron triple. The Indian mathematician Brahmagupta derived the parametric version of integer heron triangles [4-6]. In [7], Charles Fleenor illustrates the existence of Heron triangles having sides whose lengths are consecutive integers. In [8], the general problem of Heron triangles with sides in any arithmetical progression is discussed. The above results motivated us to search for different set of formulas for integer heron triangles which is the main thrust of this paper.

This paper consists of three sections 1, 2 and 3. In section 1, we illustrate the process of obtaining different set of formulas for integer heron triangles. In section 2, we present heron triangles with sides in Arithmetic progression and it seems that they are not presented earlier. Section 3 deals with the different sets of isosceles heron triangles.

II. Method Of Analysis

2.1. Section: 1 Formulas for integer heron triangles

Let the three positive integers a, b, c be the lengths of the sides BC, CA, AB respectively of the heron triangle ABC. Consider the cosine formula given by

$$a^2 = b^2 + c^2 - 2bc\cos A \tag{1.1}$$

Let $\cos A = \dfrac{\alpha}{\beta}, \quad \beta > \alpha > 0$ (1.2)

where $\beta^2 - \alpha^2 = D^2 \quad (D > 0)$ (1.3)

Substitution of (1.2) in (1.1) gives

$$2bc\alpha = \beta\left(b^2 + c^2 - a^2\right) \tag{1.4}$$

Introducing the linear transformations

$$b = 2X + 2\alpha T, \quad c = 2\beta T, \quad a = 2A \tag{1.5}$$

in (1.4), it is written as

$$A^2 = X^2 + D^2T^2 \tag{1.6}$$

which is in the form of well-known Pythagorean equation satisfied by

$$X = 2mn, \quad DT = m^2 - n^2, \; A = m^2 + n^2, \; m > n > 0 \tag{1.7}$$

Choosing $m = DM$ and $n = DN$ in (1.7), we have

$$\left.\begin{aligned} X &= 2D^2MN \\ T &= D\left(M^2 - N^2\right) \\ A &= D^2\left(M^2 + N^2\right), \quad M > N > 0 \end{aligned}\right\} \tag{1.8}$$

Substituting (1.8) in (1.5), the values of a, b, c are given by

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REAL AND GAUSSIAN INTEGER SOLUTIONS TO $x^2+y^2=2(z^2-w^2)$

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ABSTRACT

The quadratic equation with four unknowns given by $x^2+y^2=2(z^2-w^2)$ is analysed for its non-zero distinct integer solutions and Gaussian integer solutions. Different choices of solutions in real and Gaussian integers are obtained. A general formula for obtaining sequence of solutions (real and complex) based on its given solution is illustrated.

***Keywords**: Quadratic with four unknowns, real integers, Gaussian integers*

I. INTRODUCTION

Number theory is the branch of Mathematics concerned with studying the properties and relations of integers. There are number of branches of number theory of which Diophantine equation is very important. Diophantine equations are numerically rich because of their variety [1-3]. In [4-11], different patterns of integer solutions to quadratic Diophantine equation with four unknowns are discussed. In [12], Gaussian integer solutions to space Pythagorean equation are obtained. In this communication, the quadratic equation with four unknowns given by $x^2+y^2=2(z^2-w^2)$ is analysed for its non-zero distinct integer solutions and Gaussian integer solutions.

II. METHOD OF ANALYSIS

2.1 Section: A (Real integer solutions)

The quadratic equation with four unknowns to be solved is

$$x^2+y^2=2(z^2-w^2) \quad (1)$$

Introduction of the linear transformations

$$x=u+v,\quad y=u-v \quad (2)$$

in (1) leads to

$$u^2+v^2+w^2=z^2 \quad (3)$$

which is in the form of space Pythagorean equation

The choices of solutions for (3) are represented below:

i) $u=m^2-n^2-p^2+q^2$, $v=2mn-2pq$,
$w=2mp+2nq$, $z=m^2+n^2+p^2+q^2$

ii) $u=2mp+2nq$, $v=2mn-2pq$,
$w=m^2-n^2-p^2+q^2$,
$z=m^2+n^2+p^2+q^2$

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The Homogeneous Bi-quadratic Equations with Five Unknowns $x^4 - y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$

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ract: In this paper the homogeneous bi-quadratic equation with five unknowns given by $- y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$ is studied for determining its non-zero distinct integer solutions. A few resting relations between the solutions and special figurate numbers are obtained.

words: homogeneous bi-quadratic, bi-quadratic with five unknowns, integer solutions.

I. INTRODUCTION

well known that the subject of diophantine equations has aroused the interest of many mathematicians since antiquity as it rs a rich variety of fascinating problems. In particular one may refer [1-11] for various problems on bi-quadratic diophantine ations with four and five variables. In this paper the homogeneous equation of degree four with five unknowns given by $- y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$ is analysed for obtaining its non-zero distinct integer solutions.

II. NOTATIONS

$SO_n = n(2n^2 - 1)$ - Stella octangular number of rank n

$CP_{6,n} = n^3$ - Centered hexagonal pyramidal number of rank n

$PR_n = n(n+1)$ - Pronic number of rank n

$OH_n = \frac{1}{3}n(2n^2 + 1)$ - Octahedral number of rank n

$t_{3,n} = \frac{n(n+1)}{2}$ - triangular number of rank n

$CP_{n,3} = \frac{n^3 + n}{2}$ - centered triangular pyramidal number of rank n

$P_n^3 = \frac{n(n+1)(n+2)}{6}$ - Tetrahedral number of rank n

$P_n^5 = \frac{n^2(n+1)}{2}$ - Pentagonal pyramidal number of rank n

$P_n^4 = \frac{n(n+1)(2n+1)}{6}$ -square pyramidal number of rank n

III.METHOD OF ANALYSIS

homogeneous biquadratic equation to be solved is

$$- y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z \quad (1)$$

oduction of the liner transformations

$$u + v,\ y = u - v,\ z = v \quad (2)$$

1), gives

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OBSERVATIONS ON THE DIOPHANTINE EQUATION $x^2+xy+y^2=12z^2$

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Abstract

A new and different set of solutions to the ternary quadratic equation $x^2+xy+y^2=12z^2$ is obtained through the concept of geometric progression and Pythagorean equation.

Keywords: homogeneous quadratic, ternary quadratic, integer solutions.

I. INTRODUCTION

It is quite obvious that Diophantine equations are rich in variety [1,3] and occupy a remarkable position since antiquity. In particular, while searching for problems in quadratic diophantine equations, the paper [4] was noticed, wherein, the author have considered the ternary quadratic diophantine equation represented by $x^2+xy+y^2=12z^2$ for non-zero distinct integer solutions and have presented some patterns of solutions. However, it is observed that there may be some more interesting sets of solutions to considered equation which is the motivation for our present communication. Four more new and different sets of solutions to the above equation are obtained through employing the concept of geometric progression and also the most cited solution of the Pythagorean equation. As far as our knowledge goes, it seems that the above solutions have not been presented earlier.

II. METHOD OF ANALYSIS

The ternary quadratic equation under consideration is

$$x^2+xy+y^2=12z^2 \quad (1)$$

Introduction of the linear transformations

$$\left.\begin{aligned} x&=2u+6v\\ y&=2u-6v\end{aligned}\right\} \quad (2)$$

in (1) leads to

$$u^2+3v^2=z^2 \quad (3)$$

Let a,b,c be three non-zero distinct integers.

Substituting

$$\left.\begin{aligned} v&=2\alpha a\\ z&=b+3\alpha^2c\\ u&=b-3\alpha^2c\end{aligned}\right\},\alpha\succ 0 \quad (4)$$

In (3), it simplifies to $a^2=bc$ (5)

which implies that the triple (b,a,c) or (c,a,b) forms a G.P.

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Remark on the Paper Entitled Lattice Points of a Cubic Diophantine Equation $11(x+y)^2 = 4(xy+11z^3)$

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act: **In this paper, new sets of solutions to the cubic equation with three unknowns given by $11(x+y)^2 = 4xy + 44z^3$ are** nted.

ords: **Ternary cubic, Integer solutions**

I. INTRODUCTION

a search is made for cubic diophantine equations, the authors noticed a paper by Manju Somanath, J. Kannan, K. Raja [1] in they have presented lattice points of the cubic diophantine equation $11(x+y)^2 = 4xy + 44z^3$. However, there are other sting sets of solutions to the above equations that are exhibited in this paper.

II. METHOD OF ANALYSIS

ider the cubic equation with three unknowns given by

$$11(x+y)^2 = 4xy + 44z^3 \quad (1)$$

art with, the substitution

$$y = (2k-1)x \quad (2)$$

gives

$$(11k^2 - 2k + 1)x^2 = 11z^3$$

n is satisfied by

$$x = 121(11k^2 - 2k + 1)\alpha^3 \quad (3)$$

$$z = 11(11k^2 - 2k + 1)\alpha^2 \quad (4)$$

that (2) – (4) satisfies (1)

n, the substitution

$$y = 2kx \quad (5)$$

leads to

$$(44k^2 + 36k + 11)x^2 = 44z^3$$

e solutions are

$$x = 242(44k^2 + 36k + 11)\alpha^3 \quad (6)$$

$$z = 11(44k^2 + 36k + 11)\alpha^2 \quad (7)$$

, (5)-(7) satisfy (1)

er,

duction of the linear transformations

$$x = u + v,\ y = u - v,\ z = u \quad (8)$$

leads to

$$v^2 = u^2(11u - 10) \quad (9)$$

performing some algebra, it is noted that (9) is satisfied by the following two choices of u and v:

$u = 11k^2 - 2k + 1,\ v = (11k - 1)(11k^2 - 2k + 1)$

$u = 11k^2 + 2k + 1,\ v = (11k + 1)(11k^2 + 2k + 1)$

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ON SYSTEMS OF DOUBLE EQUATIONS WITH SURDS

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ABSTRACT

This paper concerns with 6 different systems of double equations involving surds to obtain their solutions in real numbers respectively.

Keywords: *System of indeterminate quadratic equations, pair of quadratic equations, system of double quadratic equation, irrational solutions.*

2010 Mathematics Subject Classification: *11D99.*

INTRODUCTION

Systems of indeterminate quadratic equations of the form $ax+c=u^2$, $bx+d=v^2$ where a, b, c, d are non-zero distinct constants, have been investigated for solutions by several authors [1, 2] and with a few possible exceptions, most of the them were primarily concerned with rational solutions. Even those existing works wherein integral solutions have been attempted, deal essentially with specific cases only and do not exhibit methods of finding integral solutions in a general form. In [3], a general form of the integral solutions to the system of equations $ax+c=u^2$, $bx+d=v^2$ where a, b, c, d are non-zero distinct constants is presented when the product ab is a square free integer whereas the product cd may or may not a square integer. For other forms of system of double diophantine equations, one may refer [4-12].

In the above references, the equations are polynomial equations with integer coefficients which motivated us to search for solutions to system of equations with surds. This communication concerns with the problem of obtaining solutions a, b in real numbers satisfying each of the system of double equations with surds represented by

i) $a\sqrt{a}+b\sqrt{b}=N$, $a\sqrt{b}+b\sqrt{a}=N-1$

ii) $a\sqrt{a}+b\sqrt{b}=N+1$, $a\sqrt{b}+b\sqrt{a}=N$

iii) $a\sqrt{a}+b\sqrt{b}=N+4$, $a\sqrt{b}+b\sqrt{a}=N$

iv) $a\sqrt{a}+b\sqrt{b}=N+24$, $a\sqrt{b}+b\sqrt{a}=N$

v) $a\sqrt{a}+b\sqrt{b}=2N+1$, $a\sqrt{b}+b\sqrt{a}=N+1$

vi) $a\sqrt{a}+b\sqrt{b}=k^2+k+1$ $a\sqrt{b}+b\sqrt{a}=3k+8$

where N is an integer. In each case, a few interesting relations among the solutions are presented.

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On Cubic Equation With Four Unknowns $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3, (s \neq \pm 1)$

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bstract: In this paper, the cubic equation with four unknown given by $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3, (s \neq \pm 1)$ is onsidered for determining its non-zero distinct integer solutions.

eywords: Cubic with four unknowns, homogeneous cubic, Integer solutions.

I. INTRODUCTION

t is well-known that there are varieties of cubic equations with four unknowns to obtain integer solutions satisfying them [1-3]. In articular, different choices of cubic equations with four unknowns are presented in [4-12]. This paper has a different choice of ubic equation with four unknowns given by $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3$, $(s \neq \pm 1)$ to obtain its infinitely many on-zero distinct integer solutions.

II. METHOD OF ANALYSIS

he cubic equation with four unknowns to be solved for its non-zero distinct integer solutions is given by

$$4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3 \quad (1)$$

ntroduction of the linear transformations

$$x=u+v, y=u-v, w=2u \quad (u \neq v \neq 0) \quad (2)$$

n (1) leads to

$$v^2=s^2u^2-\left(s^2-1\right)z^2 \quad (3)$$

gain, considering the linear transformations

$$u=X+\left(s^2-1\right)T \quad (4)$$

$$z=X+s^2T \quad (5)$$

n (3), it gives

$$X^2=\left(s^4-s^2\right)T^2+v^2 \quad (6)$$

he fundamental solution of (6) is

$$T_0=2v, X_0=\left(2s^2-1\right)v$$

o obtain the other solutions of (6), consider its pellian equation

$$X^2=\left(s^4-s^2\right)T^2+1$$

hose general solution $\left(\widetilde{T}_n, \widetilde{X}_n\right)$ is given by

$$\widetilde{X}_n=\frac{1}{2}f_n, \widetilde{T}_n=\frac{1}{2\sqrt{s^4-s^2}}g_n$$

here

$$f_n=\left(2s^2-1+2\sqrt{s^4-s^2}\right)^{n+1}+\left(2s^2-1-2\sqrt{s^4-s^2}\right)^{n+1}$$

$$g_n=\left(2s^2-1+2\sqrt{s^4-s^2}\right)^{n+1}-\left(2s^2-1-2\sqrt{s^4-s^2}\right)^{n+1}$$

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Observation on the Negative Pell Equation $y^2 = 40x^2 - 36$

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***stract:** The binary quadratic equation represented by the negative pellian $y^2 = 40x^2 - 36$ is analyzed for its distinct integer ...tions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have ...ained solutions of other choices of hyperbolas and parabolas. Also, the relations between the solutions and special figurate ...nbers are exhibited.*

...words: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions and Figurate numbers.

I. INTRODUCTION

...subject of Diophantine equation is one of the areas in Number Theory that has attracted many Mathematicians since antiquity ... it has a long history. Obviously, the Diophantine equation are rich in variety [1-3]. In particular, the binary quadratic ...phantine equation of the form $y^2 = Dx^2 - N\,(N > 0,\ D > 0\ and\ square\ free)$ is referred as the negative form of the pell equation ... related pell equation. It is worth to observe that the negative pell equation is not always solvable. For example, the ...ations $y^2 = 3x^2 - 1$, $y^2 = 7x^2 - 4$ have no integer solutions whereas $y^2 = 65x^2 - 1$, $y^2 = 202x^2 - 1$ have integer solutions. In this ...text, one may refer [4-10] for a few negative pell equations with integer solutions.

...his communication, the negative pell equation given by $y^2 = 40x^2 - 36$ is considered and analysed for its integer solutions. A ... interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions ...other choices of hyperbolas and parabolas. Also, the relations between the solutions and special figurate numbers are exhibited.

II. NOTATIONS

$t_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$ - Polygonal number of rank n with size m

$P_n^m = \frac{1}{6}n(n+1)[(m-2)n + (5-m)]$ - Pyramidal number of rank n with size m

III.METHOD OF ANALYSIS

...negative Pell equation representing hyperbola under consideration is

$$y^2 = 40x^2 - 36 \qquad (1)$$

...se smallest positive integer solution is

$$x_0 = 1,\ y_0 = 2$$

...obtain the other solutions of (1), consider the Pell equation

$$y^2 = 40x^2 + 1$$

...se general solution is given by

$$\tilde{x}_n = \frac{1}{4\sqrt{10}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(19 + 3\sqrt{40}\right)^{n+1} + \left(19 - 3\sqrt{40}\right)^{n+1}$$

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ON THE NEGATIVE PELL EQUATION $x^2 = 6y^2 - 50$

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ABSTRACT

The binary quadratic equation represented by the negative pellian $x^2 = 6y^2 - 50$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are also given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas and special Pythagorean triangle.

KEYWORDS: binary quadratic, hyperbola, parabola, pell equation, integral solutions.

1. INTRODUCTION

Diophantine equation of the form $y^2 = Dx^2 - 1$, where $D > 0$ and square free, is known as negative pell equation. In general, the general form of negative pell equation is represented by $y^2 = Dx^2 - N$, $N > 0$, $D > 0$ and square free. It is known that negative pell equations $y^2 = 3x^2 - 1, y^2 = 7x^2 - 4$ have no integer solutions whereas $y^2 = 65x^2 - 1$, $y^2 = 202x^2 - 1$ have integer solutions. It is observed that the negative pell equation do not always have integer solutions. For negative pell equations with integer solutions, one may refer [1-11].

In this communication, yet another negative pell equation given by $y^2 = 7x^2 - 14$ is considered for its non-zero distinct integer solutions. A few interesting relations among the solutions are also given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas and special Pythagorean triangle.

2. METHOD OF ANALYSIS

The negative pell equation representing hyperbola under consideration is

$$x^2 = 6y^2 - 50 \tag{1}$$

whose smallest positive integer solution is $x_0 = 2, y_0 = 3$

To obtain the other solutions of (1), consider the pell equation

$$x^2 = 6y^2 + 1$$

whose solution is given by

$$\tilde{x}_n = \frac{f_n}{2}, \tilde{y}_n = \frac{g_n}{2\sqrt{6}}$$

where $f_n = \left(5 + 2\sqrt{6}\right)^{n+1} + \left(5 - 2\sqrt{6}\right)^{n+1}$

$$g_n = \left(5 + 2\sqrt{6}\right)^{n+1} - \left(5 - 2\sqrt{6}\right)^{n+1}$$

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Remark on the paper entitled Observations on ternary quadratic equation $5x^2 + 7y^2 = 972z^2$

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Abstract: A new and different sets of solutions to the ternary quadratic equation $5x^2 + 7y^2 = 972z^2$ are obtained through the concept of geometric progression and Pythagorean equation.

1. Introduction

While making a survey of problems on ternary quadratic Diophantine equations, the article entitled "Observations on Ternary Quadratic Equation $5x^2 + 7y^2 = 972z^2$" is noticed in which the authors have presented a few patterns of integer solutions [1]. However, it is observed that there are other sets of interesting integer solutions to the considered quadratic equation with three unknowns which is the main thrust of this communication.

2. Method of Analysis

The ternary quadratic equation under consideration is

$$5x^2 + 7y^2 = 972z^2 \qquad (1)$$

Introduction of the linear transformations

$$x = X + 7T,\ y = X - 5T \qquad (2)$$

in (1) leads to

$$X^2 + 35T^2 = 81z^2 \qquad (3)$$

which is satisfied by

On the Diophantine Equation $x^2 + axy + by^2 = z^2$

M A Gopalan, S. Vidhyalakshmi & J. Shanthi

Shrimati Indira Gandhi College

ABSTRACT

A new and different set of solutions is obtained for the ternary quadratic diophantine equation $x^2 + axy + by^2 = z^2$ through representing it as a system of double equations.

Keywords: ternary quadratic, system of double equations, integer solutions.

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On the Diophantine Equation $x^2 + axy + by^2 = z^2$

M. A. Gopalan[α], S. Vidhyalakshmi[σ] & J. Shanthi[ρ]

I. ABSTRACT

A new and different set of solutions is obtained for the ternary quadratic diophantine equation $x^2 + axy + by^2 = z^2$ through representing it as a system of double equations.

Keywords: ternary quadratic, system of double equations, integer solutions.

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II. INTRODUCTION

The diophantine equation of the form $cx^2 + axy + by^2 = dz^2$ where a, b, c, d are non-zero integers has been discussed by several authors [1-3]. In [4-14], integer solutions to the above equation are presented when a, b, c, d take particular numerical values. In this communication, different sets of integer solutions to the above equation are obtained when $c = d = 1$ by representing it as a system of double equations involving trigonometric functions. It seems that they have not been presented earlier.

III. METHOD OF ANALYSIS

The diophantine equation under consideration is

$$x^2 + axy + by^2 = z^2 \tag{1}$$

The above equation is represented as the system of double equations as below:

System	1	2	3
$z+x$	$(ax+by)\sec\theta$	$y\cot\theta$	$(ax+by)\cot\theta$
$z-x$	$y\cos\theta$	$(ax+by)\tan\theta$	$y\tan\theta$

Consider system: 1

Elimination of z leads to

$$x = t(b-\cos^2\theta),\ \ y = t(2\cos\theta - a) \tag{2}$$

Case: 1

Assume

$$\cos\theta = \frac{2pq}{\sqrt{t}},\ \ t = (p^2+q^2)^2 \tag{3}$$

Substituting (3) in (2), we have

$$x = b(p^4+q^4) + 2p^2q^2(b-2)$$

$$y = (p^2+q^2)(4pq - a(p^2+q^2))$$

and from the given system

$$z = b(p^4+q^4) + 2p^2q^2(b+2) - 2apq(p^2+q^2)$$

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Research paper

On gaussian diophantine quadruples

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Abstract

This paper concerns with the problem of constructing gaussian diophantine quadruples with the property that the product of any two distinct gaussian integers added with 1 and 4 in turn is a perfect square. The construction of gaussian diophantine quadruple (A,B,C,D) is illustrated through employing the non-zero distinct integer solutions of the system of double diophantine equations. The repetition of the above process leads to the generation of sequences of gaussian diophantine quadruples with the given property.

Keywords: *Diophantine Quadruple; Double Diophantine Equations; Gaussian Diophantine Quadruples; Integer Solutions; Pell Equations.*

1. Introduction

The construction of the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus. A set of m distinct non-zero integers $\{a_1, a_2, \ldots, a_m\}$ is called a Diophantine m-tuple with property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \le i \le j \le m$ [1]. Various mathematicians discussed the construction of different formulations of Diophantine triple and diophantine quadruples with property $D(n)$ for any arbitrary integer n and also for polynomials in n [2-15].

A set $\{a_1, a_2, \ldots, a_m\} \subset Z(i) - \{0\}$ is said to have this property $D(z)$ if the product of its any two distinct elements increased by z is a square of a Gaussian integer. If the set $\{a_1, a_2, \ldots, a_m\}$ is a complex diophantine quadruple then the same is true for the set $\{-a_1, -a_2, \ldots, -a_m\}$. Particularly in [16], the authors have analyzed the problem of the existence of the complex diophantine quadruples. In this context, one may refer [17-25].

In this communication, we construct sequences of gaussian diophantine quadruples with properties $D(1)$ and $D(4)$.

2. Method of analysis

2.1. Problem 1:

Let A, B be two gaussian integers represented by

$$A = kp \pm k + ikq\ ,\ B = kp \pm (2n+k) + ikq$$

where k, p, q and n are non-zero integers.

Note that

$$AB + n^2 = \left(kp \pm (n+k) + ikq\right)^2 = r^2 \text{ (say)}$$

Therefore, the pair (A,B) is a gaussian diophantine 2-tuple with property $D(n^2)$

Consider C to be a gaussian integer such that

$$AC + n^2 = \alpha^2 \tag{1}$$

$$BC + n^2 = \beta^2 \tag{2}$$

Assume

$$\alpha = A + r\ ,\ \beta = B + r \tag{3}$$

Substituting (3) in (1) and (2) and subtracting one from the other, observe that

$$C = A + B + 2r = 4kp \pm 4(n+k) + i\,4kq$$

It is observed that the triple (A,B,C) is a gaussian diophantine 3-tuple with property $D(n^2)$. When $n=1, n=2$, the above triple (A,B,C) can be extended to diophantine quadruple with their corresponding properties.

2.1.1. Diophantine quadruple with property $D(1)$:

Let $n=1$. Then the triple (A,B,C) is given by

$$A = kp \pm k + ikq$$

$$B = kp \pm (2+k) + ikq$$

$$C = 4kp \pm 4(k+1) + i\,4kq$$

which is a gaussian diophantine triple with property $D(1)$.

If (A,B,C) is a diophantine triple with property $D(1)$ then the fourth tuple D is given by

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On the Positive Pell Equation $y^2 = 32x^2 + 41$

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Abstract: ***The binary quadratic equation represented by the positive Pellian*** $y^2 = 32x^2 + 41$ ***is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbola and parabola.***

Keywords: ***Binary quadratic, hyperbola, integral solutions, parabola, Pell equation. 2010 mathematics subject classification: 11D09***

I. INTRODUCTION

A binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-2]. For an extensive review of various problems, one may refer [3-12]. In this communication, yet another interesting hyperbola given by $y^2 = 32x^2 + 41$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are obtained.

A. Method of Analysis

The positive Pell equation representing hyperbola under consideration is

$$y^2 = 32x^2 + 41 \tag{1}$$

whose smallest positive integer solution is

$$x_0 = 2,\ y_0 = 13$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 32x^2 + 1$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{8\sqrt{2}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(17 + 12\sqrt{2}\right)^{n+1} + \left(17 - 12\sqrt{2}\right)^{n+1}$$

$$g_n = \left(17 + 12\sqrt{2}\right)^{n+1} - \left(17 - 12\sqrt{2}\right)^{n+1},\quad n = -1, 0, 1, 2......$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = f_n + \frac{13}{8\sqrt{2}} g_n$$

$$y_{n+1} = \frac{13}{2} f_n + 8\sqrt{2} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 34x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 34y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table: 1 below:

A CONNECTION BETWEEN PAIRS OF RECTANGLES AND SPHENIC NUMBERS

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Abstract : This paper aims at determining pairs of rectangles such that, in each pair, the sum of their areas is represented by a Sphenic number. Also, the number of primitive and non-primitive rectangles for each sphenic number is given.

IndexTerms - **Pairs of rectangles, Area, Sphenic number.**

I. INTRODUCTION

Any sequence of numbers represented by a mathematical function may be considered as pattern. In fact, mathematics can e considered as a characterization of patterns. For clear understanding, any regularity that can be illustrated by a scientific theory s a pattern. In other words, a pattern is a group of numbers, shapes or objects that follow a rule. A careful observer of patterns may ote that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from he above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha umbers. For illustrations, one may refer [1- 21].

II. DEFINITION

phenic Number:

A Sphenic number is a positive integer which is the product of exactly 3 distinct primes.

II. METHOD OF ANALYSIS

et $R_1(x,y)$ and $R_2(z,w)$ be two distinct rectangles whose corresponding areas are A_1, A_2.

onsider

$A_1 + A_2 = 30$, a sphenic number

hat is,

$$xy + zw = 30 \quad (1)$$

et q, r, s be three non-zero distinct positive integers and $r > s$.

troduction of the linear transformations

$$x = s\ ,\ y = 2q + s\ ,\ z = r - s\ ,\ w = r + s \quad (2)$$

(1) leads to

$$r^2 = 30 - 2qs \quad (3)$$

lving (3) for q, r, s and using (2), the corresponding values of rectangles R_1 and R_2 are obtained and presented in Table:1 low:

Table: 1 Rectangles

R_1	R_2	$A_1 + A_2$	Observations	
			Primitive	Non-Primitive
(1, 15)	(3, 5)	30	R_1, R_2	
(1, 27)	(1, 3)	30	R_1, R_2	

te that the above two pairs of rectangles are primitives as $\gcd(x,y)=1$ and $\gcd(z,w)=1$

me other numerical examples of sphenic numbers are presented in Table: 2 below:

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On the Positive Pell Equation $y^2 = 35x^2 + 14$

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***Abstract:** The binary quadratic equation represented by the positive Pellian $y^2 = 35x^2 + 14$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbola and parabola.*
***Keywords:** Binary quadratic, hyperbola, integral solutions, parabola, pell equation. 2010 mathematics subject classification: 11D09*

I. INTRODUCTION

A binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-2]. For an extensive review of various problems, one may refer [3-12]. In this communication, yet another interesting hyperbola given by $y^2 = 35x^2 + 14$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are obtained.

A. Method of Analysis

The Positive Pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 14 \qquad (1)$$

whose smallest positive integer solution is

$$x_0 = 1,\ y_0 = 7$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 35x^2 + 1$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n ,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(6+\sqrt{35}\right)^{n+1} + \left(6-\sqrt{35}\right)^{n+1}$$

$$g_n = \left(6+\sqrt{35}\right)^{n+1} - \left(6-\sqrt{35}\right)^{n+1} \quad , \ n = -1,0,1,2......$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{7}{2\sqrt{35}} g_n$$

$$y_{n+1} = \frac{7}{2} f_n + \frac{\sqrt{35}}{2} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 12x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 12y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table: 1 below:

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ON THE HEPTIC EQUATION WITH FIVE UNKNOWNS $x^4+y^4-(y+x)w^3=14z^2T^5$

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ABSTRACT

We obtain infinitely many non-zero integer quintuples (x, y, z, w, T) satisfying the non-homogeneous equation of degree seven with five unknowns given by $x^4+y^4-(y+x)w^3=14z^2T^5$. Various interesting properties between the solutions and special numbers are presented.

KEYWORDS: Higher degree, Heptic with five unknowns, Integer solutions.

1. INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, homogeneous and non-homogeneous equations of higher degree have aroused the interest of numerous Mathematicians since antiquity [1-3]. Particularly, in [4-10], heptic equations with three, four and five unknowns are analyzed. This paper concerns with yet another problem of determining non-trivial integral solutions of the non-homogeneous equation of seventh degree with five unknowns given by $x^4+y^4-(y+x)w^3=14z^2T^5$. A few relations between the solutions and the special numbers are presented.

2. NOTATIONS

- Polygonal number of rank n with size m

$$t_{m,n}=n\left[1+\frac{(n-1)(m-2)}{2}\right]$$

- Pyramidal number of rank n with size m

$$P_n^m=\frac{1}{6}[n(n+1)][(m-2)n+(5-m)]$$

- Centered Pyramidal number of rank n with size m

$$CP_{m,n}=\frac{m(n-1)n(n+1)+6n}{6}$$

- Stella Octangular number of rank n

$$SO_n=2n^3-n$$

- Gnomonic number of rank n

$$GNO_n=2n-1$$

- Pronic number of rank n

$$Pr_n=n(n+1)$$

- Five dimensional Figurate number of rank n whose generating polygon is a triangle

$$F_{5,n,3}=\frac{n^5+10n^4+35n^3+50n^2+24n}{5!}$$

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ON THE TRANSCENDENTAL EQUATION WITH THREE UNKNOWNS $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$

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ABSTRACT
The transcendental equation with three unknowns given by $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$ is considered and analyzed for finding different sets of integer solutions.

KEYWORDS: Transcendental equation, Integer solutions.

1. INTRODUCTION
The subject of diophantine equation, one of the interesting areas of Number Theory, plays a significat role in higher arithmetic and has a marvelous effect on credulous people and always occupies a remarkable position due to unquestioned historical importance. The diophantine equations may be either polynomial equation with atleast two unknowns for which integer solution, are required or transcendental equation involving trigonometric, logarithmic, exponential and surd function such that one may be interested in getting integer solution.

It seems that much work has not been done with regard to integer solution for transcendental equation with surds. In this context, one may refer [1-10].

In this paper, we are interested in obtaining integer solutions to transcendental equation involving surds. In particular, we obtain different sets of integer solutions to the transcendental equation with three unknowns given by $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$.

2. METHOD OF ANALYSIS
The ternary transcendental equation to be solved is

$$2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2 \tag{1}$$

Introduction of the transformations

$$x=(u+v)^2;\ y=(u-v)^2;\quad u\neq v\neq 0 \tag{2}$$

in (1) leads to

$$u^2+7v^2=(k^2+7s^2)z^2 \tag{3}$$

The above equation (3) is solved through different methods and using (2), one obtains different sets of solutions to (1).

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ON THE PELL-LIKE EQUATION

$3x^2 - 8y^2 = 40$

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ABSTRACT

The hyperbola represented by the binary quadratic equation $3x^2 - 8y^2 = 40$ is analyzed for finding its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented.

KEYWORDS: *Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

2010 Mathematics subject classification: 11D09

1. NOTATION

$t_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$-Polygonal number of rank n with sides m

2. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-13].

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On the hyperbola $X^2+4XY+Y^2-2X+2Y-8=0$

S. Vidhyalakshmi,[1*], M.A. Gopalan[2] and S. Aarthy Thangam[3]

Abstract

In this work, we search for the lattice points of the hyperbola $X^2+4XY+Y^2-2X+2Y-8=0$. Various connections among the solutions are given. Given a solution, solutions for other forms of hyperbolas and parabolas are determined.

Keywords

Binary quadratic, Hyperbola, Parabola, Pell equation, Integral solutions.

AMS Subject Classification

11D09.

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Contents

1. Introduction

Every student of mathematics is familiar with the subject of analytical geometry which is a study of geometry using a co-ordinate system. Linear equations involving x and y specify lines while quadratic equations specify conic sections. The hyperbola, a special conic, represented by the pell equation $y^2=Dx^2+N$ ($D>0$ and square free) for various values of D and N are studied in [6, 8, 9]. The hyperbola represented by an equation of the form $x^2+Axy+y^2+Bx=0$ is analyzed for various values of A and B in [2-5]. In [1,7], the hyperbola represented by an equation of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ is considered for particular values A,B,C,D,E and F. It seems that much work has not been done in this choice. It is therefore towards this end, the hyperbola represented by $X^2+4XY+Y^2-2X+2Y-8=0$ is considered in this paper for determining its non-zero distinct integer solutions. Employing the solutions of the given equation, integer solutions to special hyperbolas and parabolas are obtained.

2. Method of Analysis

The diophantine equation under consideration is

$$X^2+4XY+Y^2-2X+2Y-8=0 \tag{2.1}$$

It is to be noted that (1) represents a hyperbola.

Substituting

$$X=x-1\ ,\ Y=y+1 \tag{2.2}$$

in (1), we get

$$x^2+y^2+4xy-6=0 \tag{2.3}$$

Again setting

$$x=M+N\ ,\ y=M-N \tag{2.4}$$

in (3), it simplifies to the equation

$$N^2=3M^2-3 \tag{2.5}$$

whose initial solution is $M_0=2$, $N_0=3$ Now consider the fundamental positive pell equation

$$N^2=3M^2+1 \tag{2.6}$$

whose general solution $(\tilde{M}_s,\tilde{N}_s)$ is given by

$$\tilde{N}_s=\frac{1}{2}f_s\ ,\ \tilde{M}_s=\frac{1}{2\sqrt{3}}g_s$$

where

$$f_s=(2+\sqrt{3})^{s+1}+(2-\sqrt{3})^{s+1},$$
$$g_s=(2+\sqrt{3})^{s+1}-(2-\sqrt{3})^{s+1},$$
$$s=-1,0,1,2,\ldots..$$

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ON THE PAIR OF EQUATIONS $a \pm b = p^3$, $ab = q^2$

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ABSTRACT

This communication aims at determining pairs of non-zero distinct integers (a, b) such that, in each pair

(i). the sum is a cubic integer and the product is a square integer
(ii). the difference is a cubical integer and the product is a square integer

KEYWORDS: *system of double equations, integer solutions*

1. INTRODUCTION

In the history of number theory, the Diophantine equations occupy a remarkable position as it has an unlimited supply of fascinating and innovating problems [1-9]. This communication concerns with the problem of obtaining two non-zero distinct integers a and b such that

(i). $a+b=p^3$, $ab=q^2$ and

(ii). $a-b=p^3$, $ab=q^2$

2. METHOD OF ANALYSIS

(I) On the system $a+b=p^3$, $ab=q^2$

Let a,b be two non-zero distinct positive integers such that

$$a+b=p^3 \, , \; ab=q^2 \qquad (1,2)$$

where $p,q>0$

The elimination of b between (1) and (2) leads to

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Observation on the Positive Pell Equation $y^2 = 15x^2 + 10$

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Abstract: The binary quadratic Diophantine equation represented by the positive Pellian $y^2 = 15x^2 + 10$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.

Keywords: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer, has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values [1-4]. For an extensive review of various problems, one may refer [5-16]. In this communication, yet another an interesting equation given by $y^2 = 15x^2 + 10$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

II. METHOD OF ANALYSIS

The Positive Pell equation representing hyperbola under consideration is

$$y^2 = 15x^2 + 10 \tag{1}$$

The smallest positive integer solutions of (1) are

$$x_0 = 1 , y_0 = 5$$

To obtain the other solutions of (1), consider the pellian equation

$$y^2 = 15x^2 + 1 \tag{2}$$

whose initial solution is given by

$$\tilde{x}_0 = 1 , \tilde{y}_0 = 4$$

The general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{15}} g_n , \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = (4+\sqrt{15})^{n+1} + (4-\sqrt{15})^{n+1}$$

$$g_n = (4+\sqrt{15})^{n+1} - (4-\sqrt{15})^{n+1} \quad , n = -1,0,1,...$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solution of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{5}{2\sqrt{15}} g_n$$

$$y_{n+1} = \frac{5}{2} f_n + \frac{\sqrt{15}}{2} g_n$$

The recurrence relations satisfied by the solutions x and y are given by

$$x_{n+3} - 8x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 8y_{n+2} + y_{n+1} = 0$$

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Observation on the Negative Pell Equation $y^2 = 12x^2 - 23$

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***Abstract:** The binary quadratic equation represented by the negative pellian $y^2 = 12x^2 - 23$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

***Keywords:** Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions*

I. INTRODUCTION

The subject of Diophantine equation is one of the areas in Number Theory that has attracted many Mathematicians since antiquity and it has a long history. Obviously, the Diophantine equation are rich in variety [1-3]. In particular, the binary quadratic diophantine equation of the form $y^2 = Dx^2 - N$ $(N > 0,\ D > 0\ and\ square\ free)$ is referred as the negative form of the pell equation (or) related pell equation. It is worth to observe that the negative pell equation is not always solvable. For example, the equations $y^2 = 3x^2 - 1$, $y^2 = 7x^2 - 4$ have no integer solutions whereas $y^2 = 65x^2 - 1$, $y^2 = 202x^2 - 1$ have integer solutions. In this context, one may refer [4-10] for a few negative pell equations with integer solutions. In this communication, the negative pell equation given by $y^2 = 12x^2 - 23$ is considered and analysed for its integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.

II. METHOD OF ANALYSIS

The Negative Pell equation representing hyperbola under consideration is

$$y^2 = 12x^2 - 23 \tag{1}$$

The smallest positive integer solutions of (1) are

$$x_0 = 2\,,\ y_0 = 5$$

To obtain the other solutions of (1), consider the pellian equation

$$y^2 = 12x^2 + 1 \tag{2}$$

whose initial solution is given by

$$\widetilde{x}_0 = 2\,,\ \widetilde{y}_0 = 7$$

The general solution $(\widetilde{x}_n, \widetilde{y}_n)$ of (2) is given by

$$\widetilde{x}_n = \frac{1}{4\sqrt{3}} g_n\,,\ \widetilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = (7+4\sqrt{3})^{n+1} + (7-4\sqrt{3})^{n+1}$$

$$g_n = (7+4\sqrt{3})^{n+1} - (7-4\sqrt{3})^{n+1}\ ,\ n = -1, 0, 1, \ldots$$

Applying Brahmagupta lemma between the solutions (x_0, y_0) and $(\widetilde{x}_n, \widetilde{y}_n)$, the other integer solutions to (1) are given by

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On the Pellian Like Equation $5x^2 - 7y^2 = -8$

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***Abstract** - The binary quadratic equation represented by the pellian like equation $5x^2 - 7y^2 = -8$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

Key Words: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions.

1. INTRODUCTION

The binary quadratic Diophantine equation of the form $ax^2 - by^2 = N, (a,b,N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a,b and N. In this context, one may refer [1-14].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $5x^2 - 7y^2 = -8$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

2. Method of Analysis

The Diophantine Equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$5x^2 - 7y^2 = -8 \qquad (1)$$

Consider the linear transformations

$$x = X + 7T \quad y = X + 5T \qquad (2)$$

From (1) and (2), we have

$$X^2 = 35T^2 + 4 \qquad (3)$$

whose smallest positive integer solution is

$$X_0 = 12 \quad T_0 = 2$$

To obtain the other solutions of (3), consider the pellian equation is

$$X^2 = 35T^2 + 1 \qquad (4)$$

whose smallest positive integer solution is

$$(\widetilde{X}_0, \widetilde{T}_o) = (6,1)$$

The general solution of (4) is given by

$$\widetilde{T}_n = \frac{1}{2\sqrt{35}} g_n \ , \ \widetilde{X}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(6+\sqrt{35}\right)^{n+1} + \left(6-\sqrt{35}\right)^{n+1}$$
$$g_n = \left(6+\sqrt{35}\right)^{n+1} - \left(6-\sqrt{35}\right)^{n+1}$$

Applying Brahmagupta lemma between (X_0, T_0) and $(\widetilde{X}_n, \widetilde{T}_n)$ the other integer solutions of (3) are given by

$$\left.\begin{aligned} X_{n+1} &= 6f_n + \sqrt{35} g_n \\ T_{n+1} &= f_n + \frac{6}{\sqrt{35}} g_n \end{aligned}\right\} \qquad (5)$$

From (2), (4) and (5) the values of x and y satisfying (1) are given by

$$x_{n+1} = 13f_n + \frac{77}{\sqrt{35}} g_n$$

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On the Positive Pell Equation $y^2 = 21x^2 + 4$

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Abstract: *The binary quadratic Diophantine equation represented by the positive pellian $y^2 = 21x^2 + 4$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Further, employing the solutions we have obtained the solutions of other choices of hyperbolas and parabolas.*

Keywords: *Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

I. INTRODUCTION

The binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values[1-4]. For an extensive review of various problems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 21x^2 + 4$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

II. NOTATIONS

$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$ polygonal number of rank n with size m

$P_n^m = \frac{1}{6}n(n+1)((m-2)n + 5 - m)$ Pyramidal number of rank n with size m

III. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is,

$$y^2 = 21x^2 + 4 \qquad (1)$$

The smallest positive integer solutions of (1) are,

$$x_0 = 1, y_0 = 5 \quad D = 21$$

Consider the pellian equation is

$$y^2 = 21x^2 + 1 \qquad (2)$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 12, \tilde{y}_0 = 55,$$

The general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{21}} g_n, \ \tilde{y}_n = \frac{1}{2} f_n$$

Where,

$$f_n = (55 + 12\sqrt{21})^{n+1} + (55 - 12\sqrt{21})^{n+1}$$

$$g_n = (55 + 12\sqrt{21})^{n+1} - (55 - 12\sqrt{21})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

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On the Positive Pell Equation $y^2 = 17x^2 + 8$

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***Abstract:** The binary quadratic Diophantine equation $y^2 = 17x^2 + 8$ is analyzed for its non-zero distinct integral solutions. A few interesting relations among the solutions are given. Further, employing the solutions have obtained solutions of other choices of hyperbolas and parabolas.*
***Keywords:** Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

I. INTRODUCTION

The binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values[1-4] .For an extensive review of various problems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 17x^2 + 8$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented

II. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is,

$$y^2 = 17x^2 + 8 \qquad (1)$$

The smallest positive integer solutions of (1) are,

$$x_0 = 1, y_0 = 5 \quad D = 17$$

Consider the pellian equation is

$$y^2 = 17x^2 + 1 \qquad (2)$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 8, \tilde{y}_0 = 33,$$

The general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{17}} g_n, \ \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (33 + 8\sqrt{17})^{n+1} + (33 - 8\sqrt{17})^{n+1}$$

$$g_n = (33 + 8\sqrt{17})^{n+1} - (33 - 8\sqrt{17})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = \frac{1}{2} f_n + \frac{5}{2\sqrt{17}} g_n$$

$$y_{n+1} = \frac{5}{2} f_n + \frac{17}{2\sqrt{17}} g_n$$

The recurrence relation satisfied by the solution x and y are given by,

$$x_{n+3} - 66x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 66y_{n+2} + y_{n+1} = 0$$

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On the Positive Pell Equation $y^2 = 23x^2 + 13$

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***Abstract**: The binary quadratic Diophantine equation represented by the positive pellian $y^2 = 23x^2 + 13$ is analyzed for its non-zero distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas and Pythagorean triangle.*

***Keywords**: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

I. INTRODUCTION

The binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values[1-4].For an extensive review of various problems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 23x^2 + 13$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

II. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is,

$$y^2 = 23x^2 + 13 \qquad (1)$$

The smallest positive integer solutions of (1) are,

$$x_0 = 1, y_0 = 6 \quad D = 23$$

The pellian equation is

$$y^2 = 23x^2 + 1 \qquad (2)$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 5, \tilde{y}_0 = 24,$$

The general solution (x_n, y_n) of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{23}} g_n, \ \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (24 + 5\sqrt{23})^{n+1} + (24 - 5\sqrt{23})^{n+1}$$

$$g_n = (24 + 5\sqrt{23})^{n+1} - (24 - 5\sqrt{23})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = \frac{1}{2} f_n + \frac{6}{2\sqrt{23}} g_n$$

$$y_{n+1} = \frac{6}{2} f_n + \frac{23}{2\sqrt{23}} g_n$$

The recurrence relation satisfied by the solution x and y are given by,

$$x_{n+3} - 48x_{n+2} + x_{n+1} = 0$$

Observation On The Paper Entitled "Special Pairs Of Rectangles And Sphenic Number"

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Abstract:

This paper aims at presenting pairs of rectangles representing the same sphenic number where, in each pair, the sum of the areas is 2 times sphenic number -1

Keywords: Pairs of rectangles, sphenic number

2010 Mathematics Subject Classification: 11D09

Introduction:

When a search is made for collecting problems on special patterns of numbers, the article entitled "Special pairs of rectangles and sphenic number" is noticed. In the above article [1], the authors have presented only one pair of rectangles for each sphenic number. However, It seems that there are some more pairs of rectangles where, in each of the pairs, the sum of the areas is represented by 2 times sphenic number -1.

Definition:

Sphenic Number:

A Sphenic number is a positive integer which is the product of exactly 3 distinct primes.

Method of Analysis:

Let $R_1(x,y)$ and $R_2(z,w)$ be two distinct rectangles whose corresponding areas are A_1, A_2.

Consider

$$A_1 + A_2 = (2*30)-1$$

That is,

$$xy + zw = 59 \qquad (1)$$

Let q, r, s be three non-zero distinct positive integers and $r > s$.

Special Characterizations of Rectangles in Connection with Armstrong Numbers of order 3,4,5,6.

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Abstract:

This paper consists of two sections A and B. **Section A** exhibits rectangles, where, in each rectangle, the area added with its semi-perimeter is an Armstrong number with digits 3,4,5,6. **Section B** presents rectangles, where, in each rectangle, the area minus its semi-perimeter is an Armstrong number with digits 3,4,5,6.

Keywords: Rectangle, Armstrong number, Primitive rectangle, Non-Primitive rectangle.

2010 Mathematics Subject Classification: 11D99

Introduction:

In [1-15], the diophantine problems relating geometrical representations with special numbers, namely, Armstrong numbers, Sphenic numbers, Harshad numbers, etc. The above results motivated us for obtaining rectangles with special characterizations in connection with Armstrong numbers of order 3, 4, 5 and 6.

It seems that the above problems has not been considered earlier.

Definition: (Armstrong Number of Order 'n')

Let N be an n-digit number represented by

$$N = a_1.a_2.a_3.......a_n$$

If $N = a_1^n + a_2^n + a_3^n .... + a_n^n$, then N is said to be an Armstrong number of order n.

Method of Analysis:

ON BINARY QUADRATIC EQUATION $2x^2 - 3y^2 = -4$

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Abstract- The binary quadratic Diophantine equation represented by the positive Pellian $2x^2 - 3y^2 = -4$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas and Pythagorean triangle.

Key Words: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions

1. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-14].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $2x^2 - 3y^2 = -4$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

2. METHOD OF ANALYSIS:

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$2x^2 - 3y^2 = -4 \quad (1)$$

Consider the linear transformations

$$x = X + 3T,\ y = X + 2T \quad (2)$$

From (1) and (2), we have

$$X^2 = 6T^2 + 4 \quad (3)$$

Whose smallest positive integer solution is

$$X_0 = 10, T_0 = 4$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2 = 6T^2 + 1 \quad (4)$$

Whose smallest positive integer solution is $(\widetilde{X}_0, \widetilde{T}_0) = (5, 2)$ the general solution of (4) is given by

$$\widetilde{T}_n = \frac{1}{2\sqrt{6}} g_n,\ \widetilde{X}_n = \frac{1}{2} f_n$$

Where

$$f_n = (5 + 2\sqrt{6})^{n+1} + (5 - 2\sqrt{6})^{n+1}$$

$$g_n = (5 + 2\sqrt{6})^{n+1} - (5 - 2\sqrt{6})^{n+1},$$

$$n = -1, 0, 1 \ldots$$

Applying Brahmagupta lemma between $(\widetilde{x}_0, \widetilde{y}_0)$ and $(\widetilde{x}_n, \widetilde{y}_n)$ the other integer solutions of (3) are given by

$$x_{n+1} = 5f_n + \frac{12}{\sqrt{6}} g_n \quad (5)$$

$$y_{n+1} = 2f_n + \frac{5}{\sqrt{6}} g_n \quad (6)$$

From (2), (5) and (6) the values of x and y satisfying (1) are given by

$$x_{n+1} = 11f_n + \frac{27}{\sqrt{6}} g_n$$

$$y_{n+1} = 9f_n + \frac{22}{\sqrt{6}} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 10x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 10y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x_n and y_n satisfying (1) are given in the Table: 1 below.

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Integral Points on the Ternary Quadratic Diophantine Equation $y^2 = 33x^2 + 4^t, \quad t \geq 0$

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Abstract: The binary quadratic equation $y^2 = 33x^2 + 4^t$ representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of Generalized Fibonacci sequences of numbers, Generalized Lucas sequences of numbers.

Keywords: Binary quadratic integral solutions, generalized Fibonacci Sequences of numbers, generalized Lucas Sequences of numbers.

AMS Mathematics Subject Classification: 11D09

Notations

$GF_n(k,s)$: Generalized Fibonacci Sequences of rank n.

$GL_n(k,s)$: Generalized Lucas Sequences of rank n.

$$t_{m,n} = n\left(1+\frac{(n-1)(m-2)}{2}\right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m))]}{6}$$

$$\Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2}+1$$

$$S_n = 6n(n-1)+1$$

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1,2,4]. In [3] infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2 = 3x^2 + 1$. In [5] a, special Pythagorean triangle is obtained by employing the integral solutions of $y^2 = 182x^2 + 14$. In [6] different pattern of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of $y^2 = 14x^2 + 4$. In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 33x^2 + 4^t$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

II. METHODS OF ANALYSIS

Consider the binary quadratic equation

$$y^2 = 33x^2 + 4^t, \quad t \geq 0 \qquad (1)$$

with least positive integer solutions is

$$x_0 = 4(2)^t, \qquad y_0 = 23(2)^t$$

To obtain the other solutions of (1),

Observations on the Hyperbola, y2 = 14x2 + 16t, t >=0 $y^2 = 14x^2 + 16^t, t \geq 0$

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Abstract: *The binary quadratic equation $y^2 = 14x^2 + 16^t$ representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of Generalized Fibonacci sequences of numbers, Generalized Lucas sequences of numbers.*

Keywords: *Binary Quadratic Integral Solutions, Generalized Fibonacci Sequences of Numbers, Generalized Lucas Sequences of Numbers, Integral Solutions.*

2010 MS Mathematics Subject Classification:11D09

Notations

$GF_n(k,s)$: Generalized Fibonacci Sequences of rank n.

$GL_n(k,s)$: Generalized Lucas Sequences of rank n.

$$t_{m,n} = n\left(I + \frac{(n-1)(m-2)}{2}\right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m))]}{6}$$

$$\Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

$$S_n = 6n(n-1)+1$$

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D non-square positive integer has been studied by various mathematics for its non-trivial integer solutions.

When D takes different integral values [1,2,4]. In [3] infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the $y^2 = 14x^2 + 1$. In [5] a special Pythagorean triangles is obtained by employing the integral solutions of $y^2 = 182x^2 + 14^t$.

In [6] different patterns of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of $y^2 = 14x^2 + 4$. In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 14x^2 + 16^t$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

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Observation on the Binary Quadratic Equation $y^2 = 105x^2 + 4^t, t \geq 0$

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***Abstract:** The binary quadratic equation is considered and a few interesting properties among the solutions are presented.*
***Keywords:** Binary quadratic, integral solutions, Generalized Fibonacci sequences, Generalized Lucas Sequences.*

I. INTRODUCTION

The Binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-5]. In this context one may also refer [5]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation representing a hyperbola. A few interesting properties among the solutions are presented.

II. NOTATIONS

- $t_{m,n}$: Polygonal number of rank n with size m
- P_n^m : Pyramidal number of rank n with size m
- Pr_n : Pronic number of rank n
- S_n : Star number of rank n
- $Ct_{m,n}$: Centered Pyramidal number of rank n with size m
- $GF_n(k,s)$: Generalized Fibonacci sequence number of rank n
- $GL_n(k,s)$: Generalized Lucas sequence number of rank n

III.METHOD OF ANALYSIS

The binary non-homogeneous quadratic Diophantine equation represents a hyperbola to be solved for its non-zero integral solutions is

$$y^2 = 105x^2 + 4^t, t \geq 0 \tag{1}$$

The smallest positive integer solution (x_0, y_0) of (1) is

$$x_0 = 4(2^t), y_0 = 41(2^t) \tag{2}$$

To obtain the other solutions of (1), consider the pellian equation

$$y^2 = 105x^2 + 1 \tag{3}$$

Applying the Brahmagupta lemma between the solutions (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{4(2^t)}{2} f_n + \frac{41(2^t)}{2\sqrt{105}} g_n$$

$$y_{n+1} = \frac{41(2^t)}{2} f_n + \frac{420(2^t)}{2\sqrt{105}} g_n$$

N THE BINARY QUADRATIC DIOPHANTINE EQUATION $y^2 = 272x^2 + 16$

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stract - The binary quadratic equation $=272x^2+16$ is considered and a few interesting erties among the solutions are presented. Employing integral solutions of the equation under siderations a few patterns of Pythagorean triangle are erved.

y Words: Binary, Quadratic, Pyramidal numbers, integral lutions

INTRODUCTION

e binary quadratic equation of the form $y^2 = Dx^2 + 1$ ere D is non -square positive integer has been udied by various mathematicians for its non-trivial egral solutions when D takes different integral values -5]. In this context one may also refer [4,10]. These sults have motivated us to search for the integral lutions of yet another binary quadratic equation $^2=272x^2+16$ representing a hyperbola .A few eresting properties among the solution are presented. ploying the integral solutions of the equation nsideration a few patterns of Pythagorean triangles are tained.

1.1 Notations

$t_{m,n}$: Polygonal number of rank n with size m

P_n^m : Pyramidal number of rank n with size m

Pr_n : Pronic number of rank n

S_n : Star number of rank n

$CP_{m,n}$: Centered Pyramidal number of rank n with size m

$GF_n(k,s)$: Generalized Fibonacci sequence of rank n

$GL_n(k,s)$: Generalized Lucas sequence of rank n

2. METHOD OF ANALYSIS

onsider the binary quadratic Diophantine equation is

$$y^2 = 272x^2 + 16 \tag{1}$$

whose smallest positive integer solutions of (x_0, y_0) is,

$$x_0 = 8,\ y_0 = 132 \tag{2}$$

To obtain the other solutions of (1), Consider pellian equation is

$$y^2 = 272x^2 + 1 \tag{3}$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 2,\ \tilde{y}_0 = 33$$

whose general solution $(\tilde{x}_n, \tilde{y}_n)$ of (3) is given by,

$$\tilde{x}_n = \frac{1}{272} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (33 + 2\sqrt{272})^{n+1} + (33 - 272)^{n+1}$$

$$g_n = (33 + 2\sqrt{272})^{n+1} - (33 - 2\sqrt{272})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = 4f_n + \frac{66}{\sqrt{272}} g_n \tag{4}$$

$$y_{n+1} = 9f_n + \frac{40}{\sqrt{20}} g_n \tag{5}$$

Therefore (4) becomes

$$\sqrt{272}x_{n+1} = 4\sqrt{272} f_n + 66 g_n \tag{6}$$

Replace n by $n+1$ in (6), we get

$$\sqrt{272}x_{n+2} = 4\sqrt{272} f_{n+1} + 66 g_{n+1}$$

$$= 4\sqrt{272}(33 f_n + 2\sqrt{272} g_n) + 66(33 g_n + 2\sqrt{272} f_n)$$

$$\sqrt{272}x_{n+2} = 264\sqrt{272} f_n + 4354 g_n \tag{7}$$

Replace n by $n+1$ in (7), we get

INTEGRAL SOLUTIONS OF THE DIOPHANTINE EQUATION $Y^2=20x^2+4$

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Abstract: The binary quadratic equation $y^2 = 20x^2 + 4$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under considerations a few patterns of Pythagorean triangles are observed.

Key Words: Binary, Quadratic, Pyramidal numbers, integral solutions

INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non -square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values. In this context one may also refer [4, 10]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2=20x^2+4$ representing a hyperbola. A few interesting properties among the solution are presented. Employing the integral solutions of the equation consideration a few patterns of Pythagorean triangles are obtained.

Notations

- Polygonal number of rank n with size m
- Pyramidal number of rank n with size m
- Pronic number of rank n
- Star number of rank n
- Centered Pyramidal number of rank n with
-) : Generalized Fibonacci sequence of rank n
-) : Generalized Lucas sequence of rank n

METHOD OF ANALYSIS

Consider the binary quadratic Diophantine equation is

$$y^2 = 20x^2 + 4 \qquad (1)$$

Whose smallest positive integer solutions of (x_0, y_0) is,

$$x_0 = 4, y_0 = 18 \qquad (2)$$

To obtain the other solutions of (1), Consider pellian equation is

$$y^2 = 20x^2 + 1 \qquad (3)$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 2, \tilde{y}_0 = 9$$

Whose general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{20}} g_n, \ \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (9+2\sqrt{20})^{n+1} + (9-2\sqrt{20})^{n+1}$$

$$g_n = (9+2\sqrt{20})^{n+1} - (9-2\sqrt{20})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = 2f_n + \frac{9}{\sqrt{20}} g_n \qquad (4)$$

$$y_{n+1} = 9f_n + \frac{40}{\sqrt{20}} g_n \qquad (5)$$

Therefore (3) becomes

$$\sqrt{20}x_{n+1} = 2\sqrt{20} f_n + 9g_n \qquad (6)$$

Replace n by $n+1$ in (6), we get

$$\sqrt{20}x_{n+2} = 2\sqrt{20} f_{n+1} + 9g_{n+1}$$

$$= 2\sqrt{20}(9f_n + 2\sqrt{20}g_n) + 9(9g_n + 2\sqrt{20}f_n)$$

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OBSERVATION ON THE POSITIVE PELL EQUATION $y^2 = 35x^2 + 46$

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ABSTRACT

The binary quadratic equation represented by the positive pellian $y^2 = 35x^2 + 46$ is analyzed for its distinct integer solutions. A few interesting relation among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas

KEYWORDS: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solution.

1. INTRODUCTION

The binary quadratic Diophantine equations are rich is variety. The binary quadratic equation of the form $y^2 = Dx^2 + 1$.where D is non square positive integer has been satisfied by various mathematician for its non-trivial integral solution. When D takes different integral values[1-4].In [5-11] the binary quadratic non-homogeneous equation representing hyperbolas respectively are studied for their non-zero integral solutions. This communication concerns with yet another binary quadratic equation given by $y^2 = 35x^2 + 46$. The recurrence relation satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

2. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 46 \tag{1}$$

The smallest positive integer solutions of (1) are

$$x_0 = 1,\ y_0 = 9$$

To obtain the order solution of (1), consider the pellian equation

$$y^2 = 35x^2 + 1 \tag{2}$$

Whose initial solution is given by

$$\tilde{x}_0 = 1,\ \tilde{y}_0 = 6$$

The general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

Where

$$f_n = (6+\sqrt{35})^{n+1} + (6-\sqrt{35})^{n+1}$$

$$g_n = (6+\sqrt{35})^{n+1} - (6-\sqrt{35})^{n+1}, n = -1,0,1,\ldots\ldots,$$

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Observations on the Non-homogeneous binary Quadratic Equation $8x^2 - 3y^2 = 20$

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Abstract - A Non-homogeneous binary quadratic equation represents hyperbola given by $8x^2 - 3y^2 = 20$ is analyzed for its non-zero distinct integer solutions. A few interesting relation between the solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are obtained.

Key Words: **Non-homogeneous quadratic, binary quadratic, integer solutions.**

1. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, N \neq 0)$ are rich in variety and have been analysed by many mathematicians for their respective integer solutions for particular values of a , b and N. In this context, one may refer [1-3].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $8x^2 - 3y^2 = 20$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented.

2. Method of Analysis

The Diophantine equations representing the binary quadratic equation to be solved for its non-zero distinct integer solution is

$$8x^2 - 3y^2 = 20 \quad (1)$$

Consider the linear transformations

$$x = X + 3T, \; y = X + 6T \quad (2)$$

From (1) and (2), we have

$$X^2 = 30T^2 + 19 \quad (3)$$

Whose smallest positive integer solution is

$$X_0 = 7, T_0 = 1$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2 = 30T^2 + 1 \quad (4)$$

Whose smallest positive integer solution is $\left(\tilde{X}_0, \tilde{T}_0\right) = (1,5)$

On the Positive Pell Equation $y^2 = 35x^2 + 29$

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***Abstract** - The binary quadratic Diophantine equation represented by the positive Pellian $y^2 = 35x^2 + 29$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

Key Words: **Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions**

1. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer, has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values [1-4]. For an extensive review of various problems, one may refer [5-17]. In this communication, yet another an interesting equation given by $y^2 = 35x^2 + 29$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

2. METHOD OF ANALYSIS

The Positive pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 29 \tag{1}$$

The smallest positive integer solutions of (1) are

$$x_0 = 1, y_0 = 8$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 35x^2 + 1 \tag{2}$$

whose initial solution is

$$\tilde{x}_0 = 1, \tilde{y}_0 = 6$$

The general Solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n, \ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = (6+\sqrt{35})^{n+1} + (6-\sqrt{35})^{n+1}$$

$$g_n = (6+\sqrt{35})^{n+1} - (6-\sqrt{35})^{n+1}, \ n = -1, 0, 1, \ldots$$

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PYTHAGOREAN TRIANGLE WITH 2A/P+H-LEG AS A NARCISSTIC NUMBER OF ORDERS 3, 4 AND 5

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ABSTRACT

This paper concerns with the problem of attaining Pythagorean triangle where, in each Pythagorean triangle expressions $\frac{2*Area}{Perimeter}+H-a$ Leg is represented by a Narcisstic numbers

Keywords: Pythagorean triangle, primitive and non primitive triangle, Narcisstic numbers.

I. INTRODUCTION

In number theory, Pythagorean triangles have been a very big interest to various mathematicians since it is a very big treasure house to hunt for. For various types of problem and ideas on Pythagorean triangle and special number, one may refer [1-12]. In this communication, we search for pairs of Pythagorean triangle so that in each pair $\frac{2*Area}{Perimeter}+Hypotenuse-a$ Leg is a Narcisstic number.

Definition: Narcisstic Number
An n digit number which is the sum of nth power of its digits is called an n- Narcisstic number. It is also known as Armstrong number.

II. METHOD OF ANALYSIS

Let T(x,y,z) be a Pythagorean triangle where

$$x=m^2-n^2, y=2mn, z=m^2+n^2 \quad (1)$$

Denote the area, perimeter and hypotenuse of T(x,y,z) by A, P and H respectively.

$$\frac{2A}{P}+H-y=\alpha$$, a Narcisstic number of orders 3,4 and 5.

The problem under consideration is equivalent to solving the Diophantine equation

$$m(m-n)=\alpha \quad (2)$$

Given α, it is possible to attain the values of m and n satisfying (2) knowing m, n and using the (1) obtains different Pythagorean triangle, each satisfying the relation $\frac{2A}{P}+H-y=\alpha$, a Narcisstic number. A few illustrations are presented in the Tables: 1, 2, 3 below:

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A Connection between Pythagorean Triangle and Sphenic Numbers

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***Abstract:** This paper concerns with the problem of obtaining many Pythagorean triangles where, in each Pythagorean triangles, the expression $\frac{2*\text{Area}}{\text{Perimeter}} + H - a\ \text{Leg}$ is represented by a Sphenic number and Sphenic palindrome number respectively. Also, we present the number of primitive and non-primitive Triangles.*

***Keywords:** Pythagorean triangles, Sphenic numbers, Sphenic Palindrome numbers, Primitive and non-primitive triangles.*

I. INTRODUCTION

Number theory is the Queen of Mathematics. It is one of the largest and oldest branches of mathematics. We may note that there is one to one correspondence between the polygonal numbers and the sides of polygon. Apart from the above patterns of numbers, Polyhedral numbers ,Nasty numbers and Dhuruva numbers have been considered in connections with Pythagorean triangles in [1-12]. In this communication, we search for patterns of Pythagorean triangles such that, in each of which, the expression $\frac{2*\text{Area}}{\text{Perimeter}} + H - a$ Leg is represented by a Sphenic number and Sphenic palindrome number and they are exhibited in sections A and B.

II. DEFINITION

1. *Palindrome Number:* Palindrome number is one that is the same when the digits are reversed.
2. *Sphenic Number:* A Sphenic number is a positive integer which is the product of exactly three distinct prime numbers.
3. *Sphenic Palindrome Number:* A Sphenic number which is palindrome is called a Sphenic palindrome number.

III.METHOD OF ANALYSIS

Let T(x,y,z) be a Pythagorean triangle where

$$x = m^2 - n^2, y = 2mn, z = m^2 + n^2 \tag{1}$$

Denote the area, perimeter and hypotenuse of T(x,y,z) by A,P and H respectively.

Section A: $\frac{2A}{P} + H - y = \alpha$, a Sphenic number of orders 3 and 4.

The problem under consideration is mathematically equivalent to solving the Diophantine equation

$$m(m-n) = \alpha \tag{2}$$

Given α, it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation, $\frac{2A}{P} + H - y = \alpha$, a Sphenic number. It is worth to note that there are only four Pythagorean triangles as the Sphenic number is a product of exactly three distinct prime numbers, A few illustrations are presented in Table 1 below.

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On the Negative Pell Equation $y^2 = 48x^2 - 23$

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***Abstract:** The binary quadratic equation represents by negative Pellian $y^2 = 48x^2 - 23$ is analyzed for its distinct integer solutions. A few interesting relations among the solution are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

***Keywords:** Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*
2010 Mathematics subject Classification (2010):11D09

I. INTRODUCTION

The binary quadratic Diophantine equations (both homogeneous and non-homogeneous) are rich in variety. In [1-17] the binary quadratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solutions. This communication concerns with yet another binary quadratic equation given by $y^2 = 48x^2 - 23$ .The recurrence relations satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

II. METHOD OF ANALYSIS

The negative Pell equation representing hyperbola under consideration is

$$y^2 = 48x^2 - 23 \tag{1}$$

whose smallest positive integer solution is

$$x_0 = 1 \, , \, y_0 = 5$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 48x^2 + 1$$

whose smallest positive integer solution is

$$\widetilde{x}_0 = 1 \, , \, \widetilde{y}_0 = 7$$

whose general solution is given by

$$\widetilde{x}_n = \frac{1}{2\sqrt{48}} g_n \, , \, \widetilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(7+\sqrt{48}\right)^{n+1} + \left(7-\sqrt{48}\right)^{n+1}$$

$$g_n = \left(7+\sqrt{48}\right)^{n+1} - \left(7-\sqrt{48}\right)^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\widetilde{x}_n, \widetilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{5}{8\sqrt{3}} g_n$$

$$y_{n+1} = \frac{5}{2} f_n + \frac{6}{\sqrt{3}} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 14x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 14y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table: 1 below:

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ON BINARY DIOPHANTINE EQUATION

$$8x^2 - 7y^2 = k^2 + 14k - 7$$

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ABSTRACT

Non-homogeneous binary quadratic equation representing hyperbola given by $8x^2 - 7y^2 = k^2 + 14k - 7$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

Keywords: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

I. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a,b,c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $8x^2 - 7y^2 = k^2 + 14k - 7$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

II. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non- zero distinct integral solution is

$$8x^2 - 7y^2 = k^2 + 14k - 7 \tag{1}$$

Introduce the linear transformation

$$x = X + 7T, y = X + 8T \tag{2}$$

From (1) & (2) we have

$$X^2 = 56T^2 + k^2 + 14k - 7 \tag{3}$$

whose smallest positive integer solution is

$X_0 = k + 7, T_0 = 1$

To obtain the other solutions of (3), consider the pell equation,

$$X^2 = 56T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

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ON THE POSITIVE PELL EQUATION $y^2 = 34x^2 + 18$

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ABSTRACT:

The binary quadratic Diophantine equation represented by the positive pellian $y^2 = 34x^2 + 18$ is analyzed for its non-zero distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, the solutions of other choices of hyperbolas, parabolas and Pythagorean triangle are obtained.

KEYWORDS:

Binary quadratic, Hyperbola, Parabola, Integral solution, Pell equation.

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...THE NEGATIVE PELL EQUATION $y^2 = 102x^2 - 18$

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...binary quadratic Diophantine equation represented by the negative pellian $y^2 = 102x^2 - 18$ is analyzed for its ...zero distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions ...the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas and Pythagorean triangle.

...WORDS: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

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On Pairs of Rectangles and Armstrong Numbers

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Abstract

This paper concerns with the problem of obtaining pairs of rectangles, where, in each pair, the sum of the areas is represented by an Armstrong number with 3 and 4 digits respectively.

Keywords: Pairs of rectangles, Armstrong number, Primitive rectangle, Non-Primitive rectangle.

2010 Mathematics Subject Classification: 11D09

Introduction:

Number is the essence of mathematical calculations. Numbers have varieties of range and richness. Many numbers exhibit fascinating properties, they form sequences, they form patterns and so on [1-9]. A careful observer of patterns may note that there is a one to one correspondence between the numbers and the number of sides of the polygon. In particular, one may refer [10-15].

In this communication, we search for pairs of rectangles where, in each pair, the sum of the areas is represented by an Armstrong number with 3 and 4 digits respectively. The total number of primitive and non-primitive rectangles is also given.

Definition: (Armstrong Number of Order 'n')

Let N be an n-digit number represented by

$$N = a_1 a_2 a_3 \ldots\ldots a_n$$

If $N = a_1^n + a_2^n + a_3^n \ldots + a_n^n$, then N is said to be an Armstrong number of order n.

In otherwords, A number that is the sum of its own digits each raised to the power of the number of digits.

Method of Analysis:

Let $R_1(x, y)$ and $R_2(X, Y)$ be two distinct rectangles whose corresponding areas are A_1, A_2.

Consider

$$A_1 + A_2 = \alpha \text{ (Armstrong Number)}$$

That is,

$$xy + XY = \alpha \tag{1}$$

Let q, r, s be three non-zero distinct positive integers and $r > s$.

Introduction of the linear transformations

$$x = s\,,\; y = 2q + s\,,\; X = r - s\,,\; Y = r + s \tag{2}$$

in (1) leads to

$$r^2 = \alpha - 2qs \tag{3}$$

Solving (3) for q, r, s and using (2), the corresponding values of rectangles R_1 and R_2 are obtained and presented in Table:1 below:

Table: 1 Rectangles

Armstrong number	R_1	R_2	Observations		Remarks
			Primitive	Non-Primitive	
153	(1, 33)	(10, 12)	R_1	R_2	Total number of Primitive rectangles =14 Total number of non-Primitive rectangles =16
	(8, 12)	(3, 19)	R_2	R_1	
	(2, 18)	(9, 13)	R_2	R_1	
	(1, 73)	(8, 10)	R_1	R_2	

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ON BINARY DIOPHANTINE EQUATION $7x^2-5y^2=8$

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ABSTRACT

Non-homogeneous binary quadratic equation representing hyperbola given by $7x^2-5y^2=8$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented.

Keywords: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

I. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2-by^2=N,(a,b,c\neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a,b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $7x^2-5y^2=8$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

II. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$7x^2-5y^2=8 \qquad (1)$$

Introduce the linear transformation

$$x=X+5T,\ y=X+7T \qquad (2)$$

From (1) & (2) we have,

$$X^2=35T^2+4 \qquad (3)$$

whose smallest positive integer solution is

$$X_0=12,\ T_0=2$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2=35T^2+1 \qquad (4)$$

whose smallest positive integer solution is

$$\tilde{X}_0=6,\ \tilde{T}_0=1$$

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ON BINARY QUADRATIC EQUATION $y^2 = 35x^2 + 29$

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ABSTRACT

The binary quadratic equation represented by the positive pellian $y^2 = 35x^2 + 29$ is analyzed for its distinct integer solutions. A few interesting relation among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas

KEYWORDS: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

1. INTRODUCTION

The binary quadratic Diophantine equations are rich in variety. The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non square positive integer has been satisfied by various mathematician for its non-trivial integral solution. When D takes different integral values [1-4]. In [5-15] the binary quadratic non-homogeneous equation representing hyperbolas respectively are studied for their non-zero integral solutions. This communication concerns with yet another binary quadratic equation given by $y^2 = 35x^2 + 29$. The recurrence relation satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

2. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 29 \tag{1}$$

whose smallest positive integer solution is

$$x_0 = 1, y_0 = 8$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 35x^2 + 1$$

whose smallest positive integer solution is

$$\tilde{x}_0 = 1, \tilde{y}_0 = 6$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n , \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(6 + \sqrt{35}\right)^{n+1} + \left(6 - \sqrt{35}\right)^{n+1}$$

On The Negative Pell Equation $y^2 = 30x^2 - 45$

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ABSTRACT:

The binary quadratic Diophantine equation represented by the negative pellian $y^2 = 30x^2 - 45$ *is analyzed for its non-zero distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas.*

KEYWORDS: *Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

I. INTRODUCTION

The binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values[1-4].For an extensive review of various problems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 30x^2 - 45$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

II. METHOD OF ANALYSIS

The negative Pell equation representing hyperbola under consideration is

$$y^2 = 30x^2 - 45 \tag{1}$$

whose smallest positive integer solution is

$$x_0 = 3 , y_0 = 15 .$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 30x^2 + 1 \tag{2}$$

whose initial solution is given by

$$\widetilde{x}_n = 2 , \widetilde{y}_n = 11$$

The general solution $(\widetilde{x}_n, \widetilde{y}_n)$ of (2) is given by,

$$\widetilde{x}_n = \frac{1}{2\sqrt{30}} g_n , \widetilde{y}_n = \frac{1}{11} f_n$$

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ON THE NON HOMOGENEOUS BINARY QUADRATIC EQUATION $4x^2-3y^2=37$

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Abstract:
This paper deals with the problem of obtaining non-zero distinct integer solutions to the non homogeneous binary quadratic equation represented by the Pell-like equation $4x^2-3y^2=37$. Different sets of integer solutions are presented. Employing the solutions of the above equation, integer solutions for other choices of hyperbolas and parabolas are obtained. A special Pythagorean triangle is also determined.

Keywords: Non-homogeneous, binary quadratic, Pell-like equation, hyperbola, parabola, integral solutions, Special numbers.

2010 Mathematics Subject Classification: 11D09

1. INTRODUCTION

The binary quadratic Diophantine equations (both homogeneous and non homogeneous) are rich in variety [1-6]. In [7-17] the binary quadratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solutions. These results have motivated us to search for infinitely many non-zero integral solutions of still another interesting binary quadratic equation given by $4x^2-3y^2=37$. The recurrence relations satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

2. METHOD OF ANALYSIS

Consider the non homogeneous binary quadratic equation

$$4x^2-3y^2=37 \tag{1}$$

Introducing the linear transformations

$$x=X\pm 3T,\ y=X\pm 4T \tag{2}$$

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SPECIAL CHARACTERIZATIONS OF POLYGONAL NUMBERS THROUGH PELL EQUATIONS

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Abstract:
In this paper, different choices of positive and negative Pell equations are considered. Employing the non-zero integer solutions of each of the above choices of positive and negative Pell equations, the relations among the special polygonal numbers are exhibited.

Keywords: Positive Pell equation, Negative Pell equation, Polygonal numbers, Integer solutions.

2010 Mathematics Subject Classification: 11D09

1. INTRODUCTION

Every researcher in Number Theory is familiar with the subject of Diophantine equations. In fact, Number theory is the great and rich intellectual heritage of man-kind and essentially a man-made world to meet his ideals of intellectual perfection. No doubt that number is the essence of mathematical calculations and one may discover beautiful patterns in numbers. Recognizing number patterns is also an important problem solving skill. It is worth to quote the remark "There is strength in numbers, but organizing those numbers is one of the great challenges" by the mathematician John C. Mather and one may call "Mathematics as the science of patterns" as remarked by Ronald Graham.

The numbers that can be represented by a regular geometric arrangement of equally spaced points are called Figurate numbers [1]. In [2], the relations among the pairs of special m-gonal numbers generated through the solutions of the binary quadratic equation $y^2 = 2x^2 - 1$ are determined. In [3], the relations among special figurate numbers through the equation $y^2 = 10x^2 + 1$ are obtained. In [4], employing the solutions of the Pythagorean equation, the relations between the pairs of special polygonal numbers such that the difference in each pair is a perfect square is obtained. Also, Bert Miller [5] has defined a number known as Nasty number as follows: A positive integer n is a Nasty number if $n = ab = cd$ and $a+b = c-d$ or $a-b = c+d$ where a, b, c and d are non-zero distinct positive integers.

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ON BINARY QUADRATIC EQUATION $5x^2 - 6y^2 = 5$

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ABSTRACT

Non-homogeneous binary quadratic equation representing hyperbola given by $5x^2 - 6y^2 = 5$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented. Also, employing the solutions of the given equation, is constructed.

KEYWORDS: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

1. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a,b,c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $5x^2 - 6y^2 = 5$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, is constructed.

2. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non- zero distinct integral solution is

$$5x^2 - 6y^2 = 5 \tag{1}$$

Introduce the linear transformation

$$x = X + 6T,\ y = X + 5T \tag{2}$$

From (1) & (2) we have

$$X^2 = 30T^2 - 5 \tag{3}$$

whose smallest positive integer solution is

$$X_0 = 5, T_0 = 1$$

To obtain the other solutions of (3), consider the pell equation ,

$$X^2 = 30T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

$$\widetilde{X}_0 = 11, \widetilde{T}_0 = 2$$

The general solution of (4) is given by,

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On Binary Diophantine Equation $3x^2 - 5y^2 = 12$

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***Abstract:** Non-homogeneous binary quadratic equation representing hyperbola given by $3x^2 - 5y^2 = 12$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented.*
***Keywords:** Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation. AMS Mathematics subject Classification (2010):11D0*

I. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a,b,c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $3x^2 - 5y^2 = 12$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented.

II. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$3x^2 - 5y^2 = 12 \quad (1)$$

Introduce the linear transformation

$$x = X + 5T, y = X + 3T \quad (2)$$

From (1) & (2) we have,

$$X^2 = 15T^2 - 6 \quad (3)$$

whose smallest positive integer solution is

$$X_0 = 3, T_0 = 1$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2 = 15T^2 + 1 \quad (4)$$

whose smallest positive integer solution is

$$\widetilde{X}_0 = 4, \widetilde{T}_0 = 1$$

whose general solution is given by

$$\widetilde{T}_n = \frac{1}{2\sqrt{15}} g_n, \widetilde{X}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(4 + \sqrt{15}\right)^{n+1} + \left(4 - \sqrt{15}\right)^{n+1}$$

$$g_n = \left(4 + \sqrt{15}\right)^{n+1} - \left(4 - \sqrt{15}\right)^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\widetilde{x}_n, \widetilde{y}_n)$, the other integer solutions of (1) are given by

$$\sqrt{15}x_{n+1} = 4\sqrt{15}f_n + 15g_n$$

$$\sqrt{15}y_{n+1} = 3\sqrt{15}f_n + 12g_n$$

Generation Formula for Integer Solutions to Special Elliptic Paraboloids

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Abstract

Knowing a solution of ternary quadratic diophantine equation representing elliptic paraboloid, a general formula for generating sequence of solutions based on the given solution is illustrated.

Keywords

ternary quadratic, generation of solutions, elliptic paraboloid.
2010 Mathematics Subject Classification: 11D09

I. INTRODUCTION

The subject of diophantine equations in number theory has attracted many mathematicians since antiquity. It is well-known that a diophantine equation is a polynomial equation in two or more unknowns with integer coefficients for which integer solutions are required. An integer solution is a solution such that all the unknowns in the equation take integer values. An extension of ordinary integers into complex numbers is the gaussian integers. A gaussian integer is a complex number whose real and imaginary parts are both integers. It is quite obvious that diophantine equations are rich in variety and there are methods available to obtain solutions either in real integers or in gaussian integers.

A natural question that arises now is, whether a general formula for generating sequence of solutions based on the given solution can be obtained? In this context, one may refer [1-7]. The main thrust of this communication is to show that the answer to the above question is in the affirmative in the case of the following ternary quadratic diophantine equations, each representing a elliptic paraboloid.

II. METHOD OF ANALYSIS

Illustration: I

The ternary quadratic diophantine equation under consideration is

$$16x^2+9y^2=4z \tag{1}$$

Let (x_0, y_0, z_0) be any solution of (1).

The solution may be in real integers or in gausssian integers or in irrational numbers.

Let (x_1, y_1, z_1) be the second solution of (1), where

$$x_1=h_0-x_0\ ,\ y_1=h_0-y_0\ ,\ z_1=z_0+6h_0^2 \tag{2}$$

in which h_0 is an unknown to be determined.

Substitution of (2) in (1) gives

$$h_0=32x_0+18y_0 \tag{3}$$

Using (3) in (2), the second solution is given by

$$x_1=31x_0+18y_0\ ,\ y_1=32x_0+17y_0 \tag{4}$$

$$z_1=z_0+6(32x_0+18y_0)^2 \tag{5}$$

Let (x_2, y_2, z_2) be the third solution of (1), where

$$x_2=h_1-x_1\ ,\ y_2=h_1-y_1\ ,\ z_2=z_1+6h_1^2$$

in which h_1 is an unknown to be determined.

The repetition of the above process leads to

$$h_1=7^2h_0\ ,\ x_2=1537x_0+864y_0\ ,\ y_2=1536x_0+865y_0 \tag{6}$$

$$z_2=z_0+6(32x_0+18y_0)^2(1+49^2) \tag{7}$$

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Observations on Two Special Hyperbolic Paraboloids

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Abstract: Knowing a solution of ternary quadratic diophantine equation representing hyperbolic paraboloid, a general formula for generating sequence of solutions based on the given solution is illustrated.
KEYWORDS: Ternary quadratic, generation of solutions, hyperbolic paraboloid

I. INTRODUCTION

The subject of diophantine equations in number theory has attracted many mathematicians since antiquity. It is well-known that a diophantine equation is a polynomial equation in two or more unknowns with integer coefficients for which integer solutions are required. An integer solution is a solution such that all the unknowns in the equation take integer values. An extension of ordinary integers into complex numbers is the gaussian integers. A gaussian integer is a complex number whose real and imaginary parts are both integers. It is quite obvious that diophantine equations are rich in variety and there are methods available to obtain solutions either in real integers or in gaussian integers.

A natural question that arises now is, whether a general formula for generating sequence of solutions based on the given solution can be obtained? In this context, one may refer [1-7]. The main thrust of this communication is to show that the answer to the above question is in the case of the following ternary quadratic diophantine equations, each representing a hyperbolic paraboloid.

II. METHOD OF ANALYSIS

Hyperbolic Paraboloid: 1

Consider the hyperbolic paraboloid given by

$$(a+1)x^2 - ay^2 = 2z \quad (1)$$

Introduction of the linear transformations

$$x = X \pm aT \ , \ y = X \pm (a+1)T \quad (2)$$

leads to

$$X^2 = (a^2+a)T^2 + 2z$$

which is satisfied by

$$T = 4k \ , \ z = 2k^2 \Rightarrow X = 2k(2a+1)$$

In view of (2), we have

$$x = 8ka + 2k \ , 2k \ \ and \ \ y = 8ka + 6k \ , -2k \quad (3)$$

Denote the above values of x, y, z as x_0, y_0, z_0 respectively. We illustrate a process of obtaining sequence of integer solutions to the given equation based on its given solution (3).

Let (x_1, y_1, z_1) be the second solution of (1), where

$$x_1 = h - x_0 \ , \ y_1 = y_0 + h \ , \ z_1 = z_0 + h \quad (4)$$

in which h is an unknown to be determined.

Substitution of (4) in (1) gives

$$h = 2(a+1)x_0 + 2ay_0 + 2 \quad (5)$$

Using (5) in (4), the second solution (x_1, y_1, z_1) of (1) is expressed in the matrix form as

$$(x_1, y_1, z_1)^t = M(x_0, y_0, z_0)^t$$

where t is the transpose and

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On Heron Triangles

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Abstract: *Different set of formulas for integer heron triangles are obtained.*
Keywords - *Heron triangles, Heron triples, Isosceles heron triangles.*

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I. Introduction

The numbers that can be represented by a regular geometric arrangement of equally spaced points are called the polygonal numbers or Figurate numbers. Mathematicians from the days of ancient Greeks have always been interested in the properties of numbers that can be arranged as a triangle, which is a three-sided polygon. There are many different kinds of triangles of which heron triangle is one. A heron triangle is a triangle having rational side lengths and rational area [1]. One may refer [2, 3] for integer heron triangles. If a, b, c are the sides of the heron triangle then the triple (a, b, c) is known as Heron triple. The Indian mathematician Brahmagupta derived the parametric version of integer heron triangles [4-6]. In [7], Charles Fleenor illustrates the existence of Heron triangles having sides whose lengths are consecutive integers. In [8], the general problem of Heron triangles with sides in any arithmetical progression is discussed. The above results motivated us to search for different set of formulas for integer heron triangles which is the main thrust of this paper.

This paper consists of three sections 1, 2 and 3. In section 1, we illustrate the process of obtaining different set of formulas for integer heron triangles. In section 2, we present heron triangles with sides in Arithmetic progression and it seems that they are not presented earlier. Section 3 deals with the different sets of isosceles heron triangles.

II. Method Of Analysis

2.1. Section: 1 Formulas for integer heron triangles

Let the three positive integers a, b, c be the lengths of the sides BC, CA, AB respectively of the heron triangle ABC. Consider the cosine formula given by

$$a^2 = b^2 + c^2 - 2bc\cos A \quad (1.1)$$

Let $\cos A = \dfrac{\alpha}{\beta}, \quad \beta > \alpha > 0$ (1.2)

where $\beta^2 - \alpha^2 = D^2 \quad (D > 0)$ (1.3)

Substitution of (1.2) in (1.1) gives

$$2bc\alpha = \beta\left(b^2 + c^2 - a^2\right) \quad (1.4)$$

Introducing the linear transformations

$$b = 2X + 2\alpha T, \quad c = 2\beta T, \quad a = 2A \quad (1.5)$$

in (1.4), it is written as

$$A^2 = X^2 + D^2T^2 \quad (1.6)$$

which is in the form of well-known Pythagorean equation satisfied by

$$X = 2mn, \quad DT = m^2 - n^2, \; A = m^2 + n^2, \; m > n > 0 \quad (1.7)$$

Choosing $m = DM$ and $n = DN$ in (1.7), we have

$$\left.\begin{aligned} X &= 2D^2MN \\ T &= D\left(M^2 - N^2\right) \\ A &= D^2\left(M^2 + N^2\right), \quad M > N > 0 \end{aligned}\right\} \quad (1.8)$$

Substituting (1.8) in (1.5), the values of a, b, c are given by

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REAL AND GAUSSIAN INTEGER SOLUTIONS TO $x^2+y^2=2(z^2-w^2)$

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ABSTRACT

The quadratic equation with four unknowns given by $x^2+y^2=2(z^2-w^2)$ is analysed for its non-zero distinct integer solutions and Gaussian integer solutions. Different choices of solutions in real and Gaussian integers are obtained. A general formula for obtaining sequence of solutions (real and complex) based on its given solution is illustrated.

Keywords: Quadratic with four unknowns, real integers, Gaussian integers

I. INTRODUCTION

Number theory is the branch of Mathematics concerned with studying the properties and relations of integers. There are number of branches of number theory of which Diophantine equation is very important. Diophantine equations are numerically rich because of their variety [1-3]. In [4-11], different patterns of integer solutions to quadratic Diophantine equation with four unknowns are discussed. In [12], Gaussian integer solutions to space Pythagorean equation are obtained. In this communication, the quadratic equation with four unknowns given by $x^2+y^2=2(z^2-w^2)$ is analysed for its non-zero distinct integer solutions and Gaussian integer solutions.

II. METHOD OF ANALYSIS

2.1 Section: A (Real integer solutions)

The quadratic equation with four unknowns to be solved is

$$x^2+y^2=2(z^2-w^2) \quad (1)$$

Introduction of the linear transformations

$$x=u+v, \quad y=u-v \quad (2)$$

in (1) leads to

$$u^2+v^2+w^2=z^2 \quad (3)$$

which is in the form of space Pythagorean equation

The choices of solutions for (3) are represented below:

i) $u=m^2-n^2-p^2+q^2$, $v=2mn-2pq$,
$w=2mp+2nq$, $z=m^2+n^2+p^2+q^2$

ii) $u=2mp+2nq$, $v=2mn-2pq$,
$w=m^2-n^2-p^2+q^2$,
$z=m^2+n^2+p^2+q^2$

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The Homogeneous Bi-quadratic Equations with Five Unknowns $x^4 - y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$

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ract: In this paper the homogeneous bi-quadratic equation with five unknowns given by
$- y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$ is studied for determining its non-zero distinct integer solutions. A few
esting relations between the solutions and special figurate numbers are obtained.
words: homogeneous bi-quadratic, bi-quadratic with five unknowns, integer solutions.

I. INTRODUCTION

well known that the subject of diophantine equations has aroused the interest of many mathematicians since antiquity as it rs a rich variety of fascinating problems. In particular one may refer [1-11] for various problems on bi-quadratic diophantine ations with four and five variables. In this paper the homogeneous equation of degree four with five unknowns given by $- y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$ is analysed for obtaining its non-zero distinct integer solutions.

II. NOTATIONS

$SO_n = n(2n^2 - 1)$ - Stella octangular number of rank n

$CP_{6,n} = n^3$ - Centered hexagonal pyramidal number of rank n

$PR_n = n(n+1)$ - Pronic number of rank n

$OH_n = \frac{1}{3}n(2n^2 + 1)$ - Octahedral number of rank n

$t_{3,n} = \frac{n(n+1)}{2}$ - triangular number of rank n

$CP_{n,3} = \frac{n^3 + n}{2}$ - centered triangular pyramidal number of rank n

$P_n^3 = \frac{n(n+1)(n+2)}{6}$ - Tetrahedral number of rank n

$P_n^5 = \frac{n^2(n+1)}{2}$ - Pentagonal pyramidal number of rank n

$P_n^4 = \frac{n(n+1)(2n+1)}{6}$ -square pyramidal number of rank n

III.METHOD OF ANALYSIS

homogeneous biquadratic equation to be solved is

$$- y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z \qquad (1)$$

oduction of the liner transformations

$$u + v,\ y = u - v,\ z = v \qquad (2)$$

1), gives

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OBSERVATIONS ON THE DIOPHANTINE EQUATION $x^2 + xy + y^2 = 12z^2$

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Abstract

A new and different set of solutions to the ternary quadratic equation $x^2 + xy + y^2 = 12z^2$ is obtained through the concept of geometric progression and Pythagorean equation.

Keywords: homogeneous quadratic, ternary quadratic, integer solutions.

I. INTRODUCTION

It is quite obvious that Diophantine equations are rich in variety [1,3] and occupy a remarkable position since antiquity. In particular, while searching for problems in quadratic diophantine equations, the paper [4] was noticed, wherein, the author have considered the ternary quadratic diophantine equation represented by $x^2 + xy + y^2 = 12z^2$ for non-zero distinct integer solutions and have presented some patterns of solutions. However, it is observed that there may be some more interesting sets of solutions to considered equation which is the motivation for our present communication. Four more new and different sets of solutions to the above equation are obtained through employing the concept of geometric progression and also the most cited solution of the Pythagorean equation. As far as our knowledge goes, it seems that the above solutions have not been presented earlier.

II. METHOD OF ANALYSIS

The ternary quadratic equation under consideration is

$$x^2 + xy + y^2 = 12z^2 \tag{1}$$

Introduction of the linear transformations

$$\left.\begin{aligned} x &= 2u + 6v \\ y &= 2u - 6v \end{aligned}\right\} \tag{2}$$

in (1) leads to

$$u^2 + 3v^2 = z^2 \tag{3}$$

Let a, b, c be three non-zero distinct integers.

Substituting

$$\left.\begin{aligned} v &= 2\alpha a \\ z &= b + 3\alpha^2 c \\ u &= b - 3\alpha^2 c \end{aligned}\right\}, \alpha \succ 0 \tag{4}$$

In (3), it simplifies to $a^2 = bc$

which implies that the triple (b,a,c) or (c,a,b) forms a G.P. (5)

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Remark on the Paper Entitled Lattice Points of a Cubic Diophantine Equation $11(x+y)^2 = 4\left(xy+11z^3\right)$

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act: **In this paper, new sets of solutions to the cubic equation with three unknowns given by** $11(x+y)^2 = 4xy + 44z^3$ **are** nted.

rds: **Ternary cubic, Integer solutions**

I. INTRODUCTION

a search is made for cubic diophantine equations, the authors noticed a paper by Manju Somanath, J. Kannan, K. Raja [1] in they have presented lattice points of the cubic diophantine equation $11(x+y)^2 = 4xy + 44z^3$. However, there are other sting sets of solutions to the above equations that are exhibited in this paper.

II. METHOD OF ANALYSIS

ider the cubic equation with three unknowns given by

$$11(x+y)^2 = 4xy + 44z^3$$

art with, the substitution

(1)

$$y = (2k-1)x$$

gives

(2)

$$\left(11k^2 - 2k + 1\right)x^2 = 11z^3$$

n is satisfied by

$$x = 121\left(11k^2 - 2k + 1\right)\alpha^3$$

$$z = 11\left(11k^2 - 2k + 1\right)\alpha^2$$

(3)

that (2) – (4) satisfies (1)

(4)

n, the substitution

$$y = 2kx$$

leads to

(5)

$$\left(44k^2 + 36k + 11\right)x^2 = 44z^3$$

e solutions are

$$x = 242\left(44k^2 + 36k + 11\right)\alpha^3$$

$$z = 11\left(44k^2 + 36k + 11\right)\alpha^2$$

(6)

, (5)-(7) satisfy (1)

(7)

er.

duction of the linear transformations

$$x = u + v, \; y = u - v, \; z = u$$

leads to

(8)

$$v^2 = u^2(11u - 10)$$

performing some algebra, it is noted that (9) is satisfied by the following two choices of u and v:

(9)

$u = 11k^2 - 2k + 1, \; v = (11k-1)\left(11k^2 - 2k + 1\right)$

$u = 11k^2 + 2k + 1, \; v = (11k+1)\left(11k^2 + 2k + 1\right)$

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ON SYSTEMS OF DOUBLE EQUATIONS WITH SURDS

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ABSTRACT

This paper concerns with 6 different systems of double equations involving surds to obtain their solutions in real numbers respectively.

Keywords: *System of indeterminate quadratic equations, pair of quadratic equations, system of double quadratic equation, irrational solutions.*

2010 Mathematics Subject Classification: *11D99.*

INTRODUCTION

Systems of indeterminate quadratic equations of the form $ax+c=u^2$, $bx+d=v^2$ where a, b, c, d are non-zero distinct constants, have been investigated for solutions by several authors [1, 2] and with a few possible exceptions, most of the them were primarily concerned with rational solutions. Even those existing works wherein integral solutions have been attempted, deal essentially with specific cases only and do not exhibit methods of finding integral solutions in a general form. In [3], a general form of the integral solutions to the system of equations $ax+c=u^2$, $bx+d=v^2$ where a, b, c, d are non-zero distinct constants is presented when the product ab is a square free integer whereas the product cd may or may not a square integer. For other forms of system of double diophantine equations, one may refer [4-12].

In the above references, the equations are polynomial equations with integer coefficients which motivated us to search for solutions to system of equations with surds. This communication concerns with the problem of obtaining solutions a, b in real numbers satisfying each of the system of double equations with surds represented by

i) $a\sqrt{a}+b\sqrt{b}=N$, $a\sqrt{b}+b\sqrt{a}=N-1$

ii) $a\sqrt{a}+b\sqrt{b}=N+1$, $a\sqrt{b}+b\sqrt{a}=N$

iii) $a\sqrt{a}+b\sqrt{b}=N+4$, $a\sqrt{b}+b\sqrt{a}=N$

iv) $a\sqrt{a}+b\sqrt{b}=N+24$, $a\sqrt{b}+b\sqrt{a}=N$

v) $a\sqrt{a}+b\sqrt{b}=2N+1$, $a\sqrt{b}+b\sqrt{a}=N+1$

vi) $a\sqrt{a}+b\sqrt{b}=k^2+k+1$ $a\sqrt{b}+b\sqrt{a}=3k+8$

where N is an integer. In each case, a few interesting relations among the solutions are presented.

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On Cubic Equation With Four Unknowns $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3,(s\neq\pm1)$

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bstract: In this paper, the cubic equation with four unknown given by $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3,(s\neq\pm1)$ is onsidered for determining its non-zero distinct integer solutions.

eywords: Cubic with four unknowns, homogeneous cubic, Integer solutions.

I. INTRODUCTION

t is well-known that there are varieties of cubic equations with four unknowns to obtain integer solutions satisfying them [1-3]. In articular, different choices of cubic equations with four unknowns are presented in [4-12]. This paper has a different choice of ubic equation with four unknowns given by $4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3$, $(s\neq\pm1)$ to obtain its infinitely many on-zero distinct integer solutions.

II. METHOD OF ANALYSIS

he cubic equation with four unknowns to be solved for its non-zero distinct integer solutions is given by

$$4\left(x^3+y^3\right)+12\left(s^2-1\right)(x+y)z^2=\left(3s^2+1\right)w^3 \quad (1)$$

ntroduction of the linear transformations

$$x=u+v,\ y=u-v,\ w=2u\ \ (u\neq v\neq 0) \quad (2)$$

n (1) leads to

$$v^2=s^2u^2-\left(s^2-1\right)z^2 \quad (3)$$

gain, considering the linear transformations

$$u=X+\left(s^2-1\right)T \quad (4)$$

$$z=X+s^2T \quad (5)$$

n (3), it gives

$$X^2=\left(s^4-s^2\right)T^2+v^2 \quad (6)$$

he fundamental solution of (6) is

$$T_0=2v,\ X_0=\left(2s^2-1\right)v$$

o obtain the other solutions of (6), consider its pellian equation

$$X^2=\left(s^4-s^2\right)T^2+1$$

hose general solution $\left(\widetilde{T}_n,\widetilde{X}_n\right)$ is given by

$$\widetilde{X}_n=\frac{1}{2}f_n,\ \widetilde{T}_n=\frac{1}{2\sqrt{s^4-s^2}}g_n$$

here

$$f_n=\left(2s^2-1+2\sqrt{s^4-s^2}\right)^{n+1}+\left(2s^2-1-2\sqrt{s^4-s^2}\right)^{n+1}$$

$$g_n=\left(2s^2-1+2\sqrt{s^4-s^2}\right)^{n+1}-\left(2s^2-1-2\sqrt{s^4-s^2}\right)^{n+1}$$

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Observation on the Negative Pell Equation $y^2 = 40x^2 - 36$

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stract: The binary quadratic equation represented by the negative pellian $y^2 = 40x^2 - 36$ is analyzed for its distinct integer utions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have ained solutions of other choices of hyperbolas and parabolas. Also, the relations between the solutions and special figurate nbers are exhibited.

words: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions and Figurate numbers.

I. INTRODUCTION

: subject of Diophantine equation is one of the areas in Number Theory that has attracted many Mathematicians since antiquity l it has a long history. Obviously, the Diophantine equation are rich in variety [1-3]. In particular, the binary quadratic phantine equation of the form $y^2 = Dx^2 - N\ (N > 0,\ D > 0\ and\ square\ free)$ is referred as the negative form of the pell equation l related pell equation. It is worth to observe that the negative pell equation is not always solvable. For example, the ations $y^2 = 3x^2 - 1$, $y^2 = 7x^2 - 4$ have no integer solutions whereas $y^2 = 65x^2 - 1$, $y^2 = 202x^2 - 1$ have integer solutions. In this text, one may refer [4-10] for a few negative pell equations with integer solutions.

his communication, the negative pell equation given by $y^2 = 40x^2 - 36$ is considered and analysed for its integer solutions. A interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions ther choices of hyperbolas and parabolas. Also, the relations between the solutions and special figurate numbers are exhibited.

II. NOTATIONS

$t_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$ - Polygonal number of rank n with size m

$P_n^m = \frac{1}{6}n(n+1)[(m-2)n + (5-m)]$ - Pyramidal number of rank n with size m

III.METHOD OF ANALYSIS

: negative Pell equation representing hyperbola under consideration is

$$y^2 = 40x^2 - 36 \quad (1)$$

se smallest positive integer solution is

$$x_0 = 1,\ y_0 = 2$$

obtain the other solutions of (1), consider the Pell equation

$$y^2 = 40x^2 + 1$$

se general solution is given by

$$\tilde{x}_n = \frac{1}{4\sqrt{10}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(19 + 3\sqrt{40}\right)^{n+1} + \left(19 - 3\sqrt{40}\right)^{n+1}$$

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ON THE NEGATIVE PELL EQUATION $x^2 = 6y^2 - 50$

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ABSTRACT

The binary quadratic equation represented by the negative pellian $x^2 = 6y^2 - 50$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are also given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas and special Pythagorean triangle.

KEYWORDS: binary quadratic, hyperbola, parabola, pell equation, integral solutions.

1. INTRODUCTION

Diophantine equation of the form $y^2 = Dx^2 - 1$, where $D > 0$ and square free, is known as negative pell equation. In general, the general form of negative pell equation is represented by $y^2 = Dx^2 - N$, $N > 0$, $D > 0$ and square free. It is known that negative pell equations $y^2 = 3x^2 - 1, y^2 = 7x^2 - 4$ have no integer solutions whereas $y^2 = 65x^2 - 1$, $y^2 = 202x^2 - 1$ have integer solutions. It is observed that the negative pell equation do not always have integer solutions. For negative pell equations with integer solutions, one may refer [1-11].

In this communication, yet another negative pell equation given by $y^2 = 7x^2 - 14$ is considered for its non-zero distinct integer solutions. A few interesting relations among the solutions are also given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas and special Pythagorean triangle.

2. METHOD OF ANALYSIS

The negative pell equation representing hyperbola under consideration is

$$x^2 = 6y^2 - 50 \qquad (1)$$

whose smallest positive integer solution is $x_0 = 2, y_0 = 3$

To obtain the other solutions of (1), consider the pell equation

$$x^2 = 6y^2 + 1$$

whose solution is given by

$$\tilde{x}_n = \frac{f_n}{2}, \tilde{y}_n = \frac{g_n}{2\sqrt{6}}$$

where $f_n = \left(5 + 2\sqrt{6}\right)^{n+1} + \left(5 - 2\sqrt{6}\right)^{n+1}$

$$g_n = \left(5 + 2\sqrt{6}\right)^{n+1} - \left(5 - 2\sqrt{6}\right)^{n+1}$$

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Remark on the paper entitled Observations on ternary quadratic equation $5x^2 + 7y^2 = 972z^2$

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Abstract: A new and different sets of solutions to the ternary quadratic equation $5x^2 + 7y^2 = 972z^2$ are obtained through the concept of geometric progression and Pythagorean equation.

1. Introduction

While making a survey of problems on ternary quadratic Diophantine equations, the article entitled "Observations on Ternary Quadratic Equation $5x^2 + 7y^2 = 972z^2$" is noticed in which the authors have presented a few patterns of integer solutions [1]. However, it is observed that there are other sets of interesting integer solutions to the considered quadratic equation with three unknowns which is the main thrust of this communication.

2. Method of Analysis

The ternary quadratic equation under consideration is

$$5x^2 + 7y^2 = 972z^2 \tag{1}$$

Introduction of the linear transformations

$$x = X + 7T,\ y = X - 5T \tag{2}$$

in (1) leads to

$$X^2 + 35T^2 = 81z^2 \tag{3}$$

which is satisfied by

On the Diophantine Equation $x^2 + axy + by^2 = z^2$

M. A. Gopalan, S. Vidhyalakshmi & J. Shanthi

Shrimati Indira Gandhi College

ABSTRACT

new and different set of solutions is obtained for the ternary quadratic diophantine equation $+ axy + by^2 = z^2$ through representing it as a system of double equations.

Keywords: ternary quadratic, system of double equations, integer solutions.

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On the Diophantine Equation $x^2 + axy + by^2 = z^2$

M. A. Gopalan[α], S. Vidhyalakshmi[σ] & J. Shanthi[ρ]

I. ABSTRACT

A new and different set of solutions is obtained for the ternary quadratic diophantine equation $x^2 + axy + by^2 = z^2$ through representing it as a system of double equations.

Keywords: ternary quadratic, system of double equations, integer solutions.

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II. INTRODUCTION

The diophantine equation of the form $cx^2 + axy + by^2 = dz^2$ where a, b, c, d are non-zero integers has been discussed by several authors [1-3]. In [4-14], integer solutions to the above equation are presented when a, b, c, d take particular numerical values. In this communication, different sets of integer solutions to the above equation are obtained when $c = d = 1$ by representing it as a system of double equations involving trigonometric functions. It seems that they have not been presented earlier.

III. METHOD OF ANALYSIS

The diophantine equation under consideration is

$$x^2 + axy + by^2 = z^2 \tag{1}$$

The above equation is represented as the system of double equations as below:

System	1	2	3
$z+x$	$(ax+by)\sec\theta$	$y\cot\theta$	$(ax+by)\cot\theta$
$z-x$	$y\cos\theta$	$(ax+by)\tan\theta$	$y\tan\theta$

Consider system: 1

Elimination of z leads to

$$x = t\left(b - \cos^2\theta\right),\ \ y = t\left(2\cos\theta - a\right) \tag{2}$$

Case: 1

Assume

$$\cos\theta = \frac{2pq}{\sqrt{t}},\ \ t = \left(p^2+q^2\right)^2 \tag{3}$$

Substituting (3) in (2), we have

$$x = b\left(p^4+q^4\right) + 2p^2q^2(b-2)$$

$$y = \left(p^2+q^2\right)\left(4pq - a\left(p^2+q^2\right)\right)$$

and from the given system

$$z = b\left(p^4+q^4\right) + 2p^2q^2(b+2) - 2apq\left(p^2+q^2\right)$$

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Research paper

On gaussian diophantine quadruples

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Abstract

This paper concerns with the problem of constructing gaussian diophantine quadruples with the property that the product of any two distinct gaussian integers added with 1 and 4 in turn is a perfect square. The construction of gaussian diophantine quadruple (A, B, C, D) is illustrated through employing the non-zero distinct integer solutions of the system of double diophantine equations. The repetition of the above process leads to the generation of sequences of gaussian diophantine quadruples with the given property.

Keywords: Diophantine Quadruple; Double Diophantine Equations; Gaussian Diophantine Quadruples; Integer Solutions; Pell Equations.

1. Introduction

The construction of the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus. A set of m distinct non-zero integers $\{a_1, a_2, \ldots, a_m\}$ is called a Diophantine m-tuple with property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \le i \le j \le m$ [1]. Various mathematicians discussed the construction of different formulations of Diophantine triple and diophantine quadruples with property $D(n)$ for any arbitrary integer n and also for polynomials in n [2-15].

A set $\{a_1, a_2, \ldots, a_m\} \subset Z(i) - \{0\}$ is said to have this property $D(z)$ if the product of its any two distinct elements increased by z is a square of a Gaussian integer. If the set $\{a_1, a_2, \ldots, a_m\}$ is a complex diophantine quadruple then the same is true for the set $\{-a_1, -a_2, \ldots, -a_m\}$. Particularly in [16], the authors have analyzed the problem of the existence of the complex diophantine quadruples. In this context, one may refer [17-25].

In this communication, we construct sequences of gaussian diophantine quadruples with properties $D(1)$ and $D(4)$.

2. Method of analysis

2.1. Problem 1:

Let A, B be two gaussian integers represented by

$$A = kp \pm k + ikq\ ,\ B = kp \pm (2n + k) + ikq$$

where k, p, q and n are non-zero integers.

Note that

$$AB + n^2 = \left(kp \pm (n + k) + ikq\right)^2 = r^2 \text{ (say)}$$

Therefore, the pair (A, B) is a gaussian diophantine 2-tuple with property $D(n^2)$

Consider C to be a gaussian integer such that

$$AC + n^2 = \alpha^2 \tag{1}$$

$$BC + n^2 = \beta^2 \tag{2}$$

Assume

$$\alpha = A + r\ ,\ \beta = B + r \tag{3}$$

Substituting (3) in (1) and (2) and subtracting one from the other, observe that

$$C = A + B + 2r = 4kp \pm 4(n + k) + i4kq$$

It is observed that the triple (A, B, C) is a gaussian diophantine 3-tuple with property $D(n^2)$. When $n = 1, n = 2$, the above triple (A, B, C) can be extended to diophantine quadruple with their corresponding properties.

2.1.1. Diophantine quadruple with property $D(1)$:

Let $n = 1$. Then the triple (A, B, C) is given by

$$A = kp \pm k + ikq$$

$$B = kp \pm (2 + k) + ikq$$

$$C = 4kp \pm 4(k + 1) + i4kq$$

which is a gaussian diophantine triple with property $D(1)$.

If (A, B, C) is a diophantine triple with property $D(1)$ then the fourth tuple D is given by

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On the Positive Pell Equation $y^2 = 32x^2 + 41$

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***Abstract:** The binary quadratic equation represented by the positive Pellian $y^2 = 32x^2 + 41$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbola and parabola.*

***Keywords:** Binary quadratic, hyperbola, integral solutions, parabola, Pell equation. 2010 mathematics subject classification: 11D09*

I. INTRODUCTION

A binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-2]. For an extensive review of various problems, one may refer [3-12]. In this communication, yet another interesting hyperbola given by $y^2 = 32x^2 + 41$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are obtained.

A. *Method of Analysis*

The positive Pell equation representing hyperbola under consideration is

$$y^2 = 32x^2 + 41 \qquad (1)$$

whose smallest positive integer solution is

$$x_0 = 2\,,\ y_0 = 13$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 32x^2 + 1$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{8\sqrt{2}} g_n\,,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(17 + 12\sqrt{2}\right)^{n+1} + \left(17 - 12\sqrt{2}\right)^{n+1}$$

$$g_n = \left(17 + 12\sqrt{2}\right)^{n+1} - \left(17 - 12\sqrt{2}\right)^{n+1}\ ,\ n = -1, 0, 1, 2......$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = f_n + \frac{13}{8\sqrt{2}} g_n$$

$$y_{n+1} = \frac{13}{2} f_n + 8\sqrt{2} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 34x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 34y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table: 1 below:

A CONNECTION BETWEEN PAIRS OF RECTANGLES AND SPHENIC NUMBERS

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bstract : This paper aims at determining pairs of rectangles such that, in each pair, the sum of their areas is represented by a phenic number. Also, the number of primitive and non-primitive rectangles for each sphenic number is given.

ndex Terms - **Pairs of rectangles, Area, Sphenic number.**

I. INTRODUCTION

Any sequence of numbers represented by a mathematical function may be considered as pattern. In fact, mathematics can e considered as a characterization of patterns. For clear understanding, any regularity that can be illustrated by a scientific theory s a pattern. In other words, a pattern is a group of numbers, shapes or objects that follow a rule. A careful observer of patterns may ote that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from he above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha umbers. For illustrations, one may refer [1- 21].

II. DEFINITION

phenic Number:

A Sphenic number is a positive integer which is the product of exactly 3 distinct primes.

II. METHOD OF ANALYSIS

et $R_1(x,y)$ and $R_2(z,w)$ be two distinct rectangles whose corresponding areas are A_1, A_2.

onsider

$A_1 + A_2 = 30$, a sphenic number

hat is,

$$xy + zw = 30 \tag{1}$$

et q, r, s be three non-zero distinct positive integers and $r > s$.

troduction of the linear transformations

$$x = s\ ,\ y = 2q + s\ ,\ z = r - s\ ,\ w = r + s \tag{2}$$

(1) leads to

$$r^2 = 30 - 2qs \tag{3}$$

lving (3) for q, r, s and using (2), the corresponding values of rectangles R_1 and R_2 are obtained and presented in Table:1 low:

Table: 1 Rectangles

R_1	R_2	$A_1 + A_2$	Observations	
			Primitive	Non-Primitive
(1, 15)	(3, 5)	30	R_1, R_2	
(1, 27)	(1, 3)	30	R_1, R_2	

te that the above two pairs of rectangles are primitives as $\gcd(x,y) = 1$ and $\gcd(z,w) = 1$

me other numerical examples of sphenic numbers are presented in Table: 2 below:

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On the Positive Pell Equation $y^2 = 35x^2 + 14$

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Abstract: ***The binary quadratic equation represented by the positive Pellian $y^2 = 35x^2 + 14$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbola and parabola.***
Keywords: ***Binary quadratic, hyperbola, integral solutions, parabola, pell equation. 2010 mathematics subject classification: 11D09***

I. INTRODUCTION

A binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-2]. For an extensive review of various problems, one may refer [3-12]. In this communication, yet another interesting hyperbola given by $y^2 = 35x^2 + 14$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are obtained.

A. Method of Analysis

The Positive Pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 14 \tag{1}$$

whose smallest positive integer solution is

$$x_0 = 1,\ y_0 = 7$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 35x^2 + 1$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(6+\sqrt{35}\right)^{n+1} + \left(6-\sqrt{35}\right)^{n+1}$$

$$g_n = \left(6+\sqrt{35}\right)^{n+1} - \left(6-\sqrt{35}\right)^{n+1},\quad n = -1,0,1,2......$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{7}{2\sqrt{35}} g_n$$

$$y_{n+1} = \frac{7}{2} f_n + \frac{\sqrt{35}}{2} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 12x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 12y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table: 1 below:

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INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

ON THE HEPTIC EQUATION WITH FIVE UNKNOWNS $x^4+y^4-(y+x)w^3=14z^2T^5$

S.Vidhyalakshmi[1], M.A.Gopalan[2] & S. Aarthy Thangam*[3]

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ABSTRACT

We obtain infinitely many non-zero integer quintuples (x, y, z, w, T) satisfying the non-homogeneous equation of degree seven with five unknowns given by $x^4+y^4-(y+x)w^3=14z^2T^5$. Various interesting properties between the solutions and special numbers are presented.

KEYWORDS: Higher degree, Heptic with five unknowns, Integer solutions.

1. INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, homogeneous and non-homogeneous equations of higher degree have aroused the interest of numerous Mathematicians since antiquity [1-3]. Particularly, in [4-10], heptic equations with three, four and five unknowns are analyzed. This paper concerns with yet another problem of determining non-trivial integral solutions of the non-homogeneous equation of seventh degree with five unknowns given by $x^4+y^4-(y+x)w^3=14z^2T^5$. A few relations between the solutions and the special numbers are presented.

2. NOTATIONS

- Polygonal number of rank n with size m

$$t_{m,n}=n\left[1+\frac{(n-1)(m-2)}{2}\right]$$

- Pyramidal number of rank n with size m

$$P_n^m=\frac{1}{6}[n(n+1)][(m-2)n+(5-m)]$$

- Centered Pyramidal number of rank n with size m

$$CP_{m,n}=\frac{m(n-1)n(n+1)+6n}{6}$$

- Stella Octangular number of rank n

$$SO_n=2n^3-n$$

- Gnomonic number of rank n

$$GNO_n=2n-1$$

- Pronic number of rank n

$$Pr_n=n(n+1)$$

- Five dimensional Figurate number of rank n whose generating polygon is a triangle

$$F_{5,n,3}=\frac{n^5+10n^4+35n^3+50n^2+24n}{5!}$$

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ON THE TRANSCENDENTAL EQUATION WITH THREE UNKNOWNS $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$

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ABSTRACT

The transcendental equation with three unknowns given by $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$ is considered and analyzed for finding different sets of integer solutions.

KEYWORDS: Transcendental equation, Integer solutions.

1. INTRODUCTION

The subject of diophantine equation, one of the interesting areas of Number Theory, plays a significat role in higher arithmetic and has a marvelous effect on credulous people and always occupies a remarkable position due to unquestioned historical importance. The diophantine equations may be either polynomial equation with atleast two unknowns for which integer solution, are required or transcendental equation involving trigonometric, logarithmic, exponential and surd function such that one may be interested in getting integer solution.

It seems that much work has not been done with regard to integer solution for transcendental equation with surds. In this context, one may refer [1-10].

In this paper, we are interested in obtaining integer solutions to transcendental equation involving surds. In particular, we obtain different sets of integer solutions to the transcendental equation with three unknowns given by $2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2$.

2. METHOD OF ANALYSIS

The ternary transcendental equation to be solved is

$$2(x+y)-3\sqrt{xy}=(k^2+7s^2)z^2 \tag{1}$$

Introduction of the transformations

$$x=(u+v)^2;\ y=(u-v)^2;\quad u\neq v\neq 0 \tag{2}$$

in (1) leads to

$$u^2+7v^2=(k^2+7s^2)z^2 \tag{3}$$

The above equation (3) is solved through different methods and using (2), one obtains different sets of solutions to (1).

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ON THE PELL-LIKE EQUATION

$3x^2 - 8y^2 = 40$

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ABSTRACT

The hyperbola represented by the binary quadratic equation $3x^2 - 8y^2 = 40$ is analyzed for finding its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented.

KEYWORDS: *Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

2010 Mathematics subject classification: 11D09

1. NOTATION

$t_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$ -Polygonal number of rank n with sides m

2. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a,b,N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-13].

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On the hyperbola $X^2+4XY+Y^2-2X+2Y-8=0$

S. Vidhyalakshmi,[1*], M.A. Gopalan[2] and S. Aarthy Thangam[3]

Abstract
In this work, we search for the lattice points of the hyperbola $X^2+4XY+Y^2-2X+2Y-8=0$. Various connections among the solutions are given. Given a solution, solutions for other forms of hyperbolas and parabolas are determined.

Keywords
Binary quadratic, Hyperbola, Parabola, Pell equation, Integral solutions.

AMS Subject Classification
11D09.

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Contents

1. Introduction

Every student of mathematics is familiar with the subject of analytical geometry which is a study of geometry using a co-ordinate system. Linear equations involving x and y specify lines while quadratic equations specify conic sections. The hyperbola, a special conic, represented by the pell equation $y^2=Dx^2+N$ ($D>0$ and square free) for various values of D and N are studied in [6, 8, 9]. The hyperbola represented by an equation of the form $x^2+Axy+y^2+Bx=0$ is analyzed for various values of A and B in [2-5]. In [1,7], the hyperbola represented by an equation of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ is considered for particular values A,B,C,D,E and F. It seems that much work has not been done in this choice. It is therefore towards this end, the hyperbola represented by $X^2+4XY+Y^2-2X+2Y-8=0$ is considered in this paper for determining its non-zero distinct integer solutions. Employing the solutions of the given equation, integer solutions to special hyperbolas and parabolas are obtained.

2. Method of Analysis

The diophantine equation under consideration is

$$X^2+4XY+Y^2-2X+2Y-8=0 \tag{2.1}$$

It is to be noted that (1) represents a hyperbola.
Substituting

$$X=x-1\ ,\ Y=y+1 \tag{2.2}$$

in (1), we get

$$x^2+y^2+4xy-6=0 \tag{2.3}$$

Again setting

$$x=M+N\ ,\ y=M-N \tag{2.4}$$

in (3), it simplifies to the equation

$$N^2=3M^2-3 \tag{2.5}$$

whose initial solution is $M_0=2$, $N_0=3$ Now consider the fundamental positive pell equation

$$N^2=3M^2+1 \tag{2.6}$$

whose general solution $(\tilde{M}_s,\tilde{N}_s)$ is given by

$$\tilde{N}_s=\frac{1}{2}f_s\ ,\ \tilde{M}_s=\frac{1}{2\sqrt{3}}g_s$$

where

$$f_s=(2+\sqrt{3})^{s+1}+(2-\sqrt{3})^{s+1},$$
$$g_s=(2+\sqrt{3})^{s+1}-(2-\sqrt{3})^{s+1},$$
$$s=-1,0,1,2,\ldots..$$

EPRA International Journal of
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ON THE PAIR OF EQUATIONS

$a \pm b = p^3$, $ab = q^2$

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ABSTRACT

This communication aims at determining pairs of non-zero distinct integers (a,b) such that, in each pair

(i). the sum is a cubic integer and the product is a square integer
(ii). the difference is a cubical integer and the product is a square integer

KEYWORDS: *system of double equations, integer solutions*

1. INTRODUCTION

In the history of number theory, the Diophantine equations occupy a remarkable position as it has an unlimited supply of fascinating and innovating problems [1-9]. This communication concerns with the problem of obtaining two non-zero distinct integers a and b such that

(i). $a+b=p^3$, $ab=q^2$ and

(ii). $a-b=p^3$, $ab=q^2$

2. METHOD OF ANALYSIS

(I) On the system $a+b=p^3$, $ab=q^2$

Let a,b be two non-zero distinct positive integers such that

$$a+b=p^3 \; , \; ab=q^2 \tag{1,2}$$

where $p,q>0$

The elimination of b between (1) and (2) leads to

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Observation on the Positive Pell Equation $y^2 = 15x^2 + 10$

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***Abstract:** The binary quadratic Diophantine equation represented by the positive Pellian $y^2 = 15x^2 + 10$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

***Keywords:** Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions*

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer, has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values [1-4]. For an extensive review of various problems, one may refer [5-16]. In this communication, yet another an interesting equation given by $y^2 = 15x^2 + 10$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

II. METHOD OF ANALYSIS

The Positive Pell equation representing hyperbola under consideration is

$$y^2 = 15x^2 + 10 \qquad (1)$$

The smallest positive integer solutions of (1) are

$$x_0 = 1,\ y_0 = 5$$

To obtain the other solutions of (1), consider the pellian equation

$$y^2 = 15x^2 + 1 \qquad (2)$$

whose initial solution is given by

$$\tilde{x}_0 = 1,\ \tilde{y}_0 = 4$$

The general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{15}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = (4+\sqrt{15})^{n+1} + (4-\sqrt{15})^{n+1}$$

$$g_n = (4+\sqrt{15})^{n+1} - (4-\sqrt{15})^{n+1} \quad ,n = -1,0,1,\ldots$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solution of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{5}{2\sqrt{15}} g_n$$

$$y_{n+1} = \frac{5}{2} f_n + \frac{\sqrt{15}}{2} g_n$$

The recurrence relations satisfied by the solutions x and y are given by

$$x_{n+3} - 8x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 8y_{n+2} + y_{n+1} = 0$$

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Observation on the Negative Pell Equation $y^2 = 12x^2 - 23$

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Abstract: ***The binary quadratic equation represented by the negative pellian $y^2 = 12x^2 - 23$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.***

Keywords: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions

I. INTRODUCTION

The subject of Diophantine equation is one of the areas in Number Theory that has attracted many Mathematicians since antiquity and it has a long history. Obviously, the Diophantine equation are rich in variety [1-3]. In particular, the binary quadratic diophantine equation of the form $y^2 = Dx^2 - N\left(N > 0,\ D > 0\ and\ square\ free\right)$ is referred as the negative form of the pell equation (or) related pell equation. It is worth to observe that the negative pell equation is not always solvable. For example, the equations $y^2 = 3x^2 - 1$, $y^2 = 7x^2 - 4$ have no integer solutions whereas $y^2 = 65x^2 - 1$, $y^2 = 202x^2 - 1$ have integer solutions. In this context, one may refer [4-10] for a few negative pell equations with integer solutions. In this communication, the negative pell equation given by $y^2 = 12x^2 - 23$ is considered and analysed for its integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.

II. METHOD OF ANALYSIS

The Negative Pell equation representing hyperbola under consideration is

$$y^2 = 12x^2 - 23 \tag{1}$$

The smallest positive integer solutions of (1) are

$$x_0 = 2\,,\ y_0 = 5$$

To obtain the other solutions of (1), consider the pellian equation

$$y^2 = 12x^2 + 1 \tag{2}$$

whose initial solution is given by

$$\tilde{x}_0 = 2\,,\ \tilde{y}_0 = 7$$

The general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{4\sqrt{3}} g_n\,,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = (7 + 4\sqrt{3})^{n+1} + (7 - 4\sqrt{3})^{n+1}$$

$$g_n = (7 + 4\sqrt{3})^{n+1} - (7 - 4\sqrt{3})^{n+1}\ ,\ n = -1, 0, 1, \ldots$$

Applying Brahmagupta lemma between the solutions (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions to (1) are given by

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On the Pellian Like Equation $5x^2 - 7y^2 = -8$

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***Abstract** - The binary quadratic equation represented by the pellian like equation $5x^2 - 7y^2 = -8$ is analyzed for its distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

Key Words: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions.

1. INTRODUCTION

The binary quadratic Diophantine equation of the form $ax^2 - by^2 = N, (a,b,N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-14].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $5x^2 - 7y^2 = -8$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

2. Method of Analysis

The Diophantine Equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$5x^2 - 7y^2 = -8 \tag{1}$$

Consider the linear transformations

$$x = X + 7T \quad y = X + 5T \tag{2}$$

From (1) and (2), we have

$$X^2 = 35T^2 + 4 \tag{3}$$

whose smallest positive integer solution is

$$X_0 = 12 \quad T_0 = 2$$

To obtain the other solutions of (3), consider the pellian equation is

$$X^2 = 35T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

$$(\widetilde{X}_0, \widetilde{T}_o) = (6,1)$$

The general solution of (4) is given by

$$\widetilde{T}_n = \frac{1}{2\sqrt{35}} g_n \ , \ \ \widetilde{X}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(6+\sqrt{35}\right)^{n+1} + \left(6-\sqrt{35}\right)^{n+1}$$
$$g_n = \left(6+\sqrt{35}\right)^{n+1} - \left(6-\sqrt{35}\right)^{n+1}$$

Applying Brahmagupta lemma between (X_0, T_0) and $(\widetilde{X}_n, \widetilde{T}_n)$ the other integer solutions of (3) are given by

$$\left.\begin{aligned} X_{n+1} &= 6f_n + \sqrt{35} g_n \\ T_{n+1} &= f_n + \frac{6}{\sqrt{35}} g_n \end{aligned}\right\} \tag{5}$$

From (2), (4) and (5) the values of x and y satisfying (1) are given by

$$x_{n+1} = 13 f_n + \frac{77}{\sqrt{35}} g_n$$

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On the Positive Pell Equation $y^2 = 21x^2 + 4$

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***Abstract**: The binary quadratic Diophantine equation represented by the positive pellian $y^2 = 21x^2 + 4$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Further, employing the solutions we have obtained the solutions of other choices of hyperbolas and parabolas.*

***Keywords**: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

I. INTRODUCTION

The binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values[1-4].For an extensive review of various problems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 21x^2 + 4$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

II. NOTATIONS

$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$ polygonal number of rank n with size m

$P_n^m = \frac{1}{6}n(n+1)((m-2)n + 5 - m)$ Pyramidal number of rank n with size m

III. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is,

$$y^2 = 21x^2 + 4 \qquad (1)$$

The smallest positive integer solutions of (1) are,

$$x_0 = 1, y_0 = 5 \quad D = 21$$

Consider the pellian equation is

$$y^2 = 21x^2 + 1 \qquad (2)$$

The initial solution of pellian equation is

$$\widetilde{x}_0 = 12, \widetilde{y}_0 = 55,$$

The general solution $(\widetilde{x}_n, \widetilde{y}_n)$ of (2) is given by,

$$\widetilde{x}_n = \frac{1}{2\sqrt{21}} g_n, \ \widetilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (55 + 12\sqrt{21})^{n+1} + (55 - 12\sqrt{21})^{n+1}$$

$$g_n = (55 + 12\sqrt{21})^{n+1} - (55 - 12\sqrt{21})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\widetilde{x}_n, \widetilde{y}_n)$ the other integer solution of (1) are given by,

On the Positive Pell Equation $y^2 = 17x^2 + 8$

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...ant Professor, [2]M. Phil Scholar, Department of Mathematics, Shrimati indira Gandhi college, Trichy-620002, Tamilnadu, India.

...act: The binary quadratic Diophantine equation $y^2 = 17x^2 + 8$ is analyzed for its non-zero distinct integral solutions. A ...eresting relations among the solutions are given. Further, employing the solutions have obtained solutions of other ...es of hyperbolas and parabolas.

...rds: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

I. INTRODUCTION

...binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various ...ematicians for its non-trivial integer solutions when D takes different integral values[1-4] .For an extensive review of various ...ems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 17x^2 + 8$ is considered and ...tely many integer solutions are obtained. A few interesting properties among the solutions are presented

II. METHOD OF ANALYSIS

...positive Pell equation representing hyperbola under consideration is,

$$y^2 = 17x^2 + 8 \qquad (1)$$

...smallest positive integer solutions of (1) are,

$$x_0 = 1, y_0 = 5 \quad D = 17$$

...ider the pellian equation is

$$y^2 = 17x^2 + 1 \qquad (2)$$

...initial solution of pellian equation is

$$\tilde{x}_0 = 8, \tilde{y}_0 = 33,$$

...general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{17}} g_n, \ \tilde{y}_n = \frac{1}{2} f_n$$

...re,

$$f_n = (33 + 8\sqrt{17})^{n+1} + (33 - 8\sqrt{17})^{n+1}$$

$$g_n = (33 + 8\sqrt{17})^{n+1} - (33 - 8\sqrt{17})^{n+1}$$

...ying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = \frac{1}{2} f_n + \frac{5}{2\sqrt{17}} g_n$$

$$y_{n+1} = \frac{5}{2} f_n + \frac{17}{2\sqrt{17}} g_n$$

...ecurrence relation satisfied by the solution x and y are given by,

$$x_{n+3} - 66x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 66y_{n+2} + y_{n+1} = 0$$

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)n the Positive Pell Equation $y^2 = 23x^2 + 13$

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ract: The binary quadratic Diophantine equation represented by the positive pellian $y^2 = 23x^2 + 13$ is analyzed for its non- distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above rbola, we have obtained solutions of other choices of hyperbolas, parabolas and Pythagorean triangle.

words: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

I. INTRODUCTION

e binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various hematicians for its non-trivial integer solutions when D takes different integral values[1-4].For an extensive review of various blems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 23x^2 + 13$ is considered infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

II. METHOD OF ANALYSIS

positive Pell equation representing hyperbola under consideration is,

$$y^2 = 23x^2 + 13 \qquad (1)$$

smallest positive integer solutions of (1) are,

$$x_0 = 1, y_0 = 6 \quad D = 23$$

pellian equation is

$$y^2 = 23x^2 + 1 \qquad (2)$$

initial solution of pellian equation is

$$\tilde{x}_0 = 5, \tilde{y}_0 = 24,$$

general solution (x_n, y_n) of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{23}} g_n, \; \tilde{y}_n = \frac{1}{2} f_n$$

re,

$$f_n = (24+5\sqrt{23})^{n+1} + (24-5\sqrt{23})^{n+1}$$

$$g_n = (24+5\sqrt{23})^{n+1} - (24-5\sqrt{23})^{n+1}$$

ying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = \frac{1}{2} f_n + \frac{6}{2\sqrt{23}} g_n$$

$$y_{n+1} = \frac{6}{2} f_n + \frac{23}{2\sqrt{23}} g_n$$

The recurrence relation satisfied by the solution x and y are given by,

$$x_{n+3} - 48x_{n+2} + x_{n+1} = 0$$

Observation On The Paper Entitled "Special Pairs Of Rectangles And Sphenic Number"

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Abstract:

This paper aims at presenting pairs of rectangles representing the same sphenic number where, in each pair, the sum of the areas is 2 times sphenic number -1

Keywords: Pairs of rectangles, sphenic number

2010 Mathematics Subject Classification: 11D09

Introduction:

When a search is made for collecting problems on special patterns of numbers, the article entitled "Special pairs of rectangles and sphenic number" is noticed. In the above article [1], the authors have presented only one pair of rectangles for each sphenic number. However, It seems that there are some more pairs of rectangles where, in each of the pairs, the sum of the areas is represented by 2 times sphenic number -1.

Definition:

Sphenic Number:

A Sphenic number is a positive integer which is the product of exactly 3 distinct primes.

Method of Analysis:

Let $R_1(x,y)$ and $R_2(z,w)$ be two distinct rectangles whose corresponding areas are A_1, A_2.

Consider

$$A_1 + A_2 = (2*30)-1$$

That is,

$$xy + zw = 59 \tag{1}$$

Let q, r, s be three non-zero distinct positive integers and $r > s$.

Special Characterizations of Rectangles in Connection with Armstrong Numbers of order 3,4,5,6.

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Abstract:

This paper consists of two sections A and B. **Section A** exhibits rectangles, where, in each rectangle, the area added with its semi-perimeter is an Armstrong number with digits 3,4,5,6. **Section B** presents rectangles, where, in each rectangle, the area minus its semi-perimeter is an Armstrong number with digits 3,4,5,6.

Keywords: Rectangle, Armstrong number, Primitive rectangle, Non-Primitive rectangle.

2010 Mathematics Subject Classification: 11D99

Introduction:

In [1-15], the diophantine problems relating geometrical representations with special numbers, namely, Armstrong numbers, Sphenic numbers, Harshad numbers, etc. The above results motivated us for obtaining rectangles with special characterizations in connection with Armstrong numbers of order 3, 4, 5 and 6.

It seems that the above problems has not been considered earlier.

Definition: (Armstrong Number of Order 'n')

Let N be an n-digit number represented by

$$N = a_1.a_2.a_3.......a_n$$

If $N = a_1^n + a_2^n + a_3^n.... + a_n^n$, then N is said to be an Armstrong number of order n.

Method of Analysis:

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ON BINARY QUADRATIC EQUATION $2x^2 - 3y^2 = -4$

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Abstract- The binary quadratic Diophantine equation represented by the positive Pellian $2x^2 - 3y^2 = -4$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas and Pythagorean triangle.

Key Words: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions

1. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a,b,N \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a,b and N. In this context, one may refer [1-14].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $2x^2 - 3y^2 = -4$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

2. METHOD OF ANALYSIS:

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$2x^2 - 3y^2 = -4 \qquad (1)$$

Consider the linear transformations

$$x = X + 3T, \; y = X + 2T \qquad (2)$$

From (1) and (2), we have

$$X^2 = 6T^2 + 4 \qquad (3)$$

Whose smallest positive integer solution is

$$X_0 = 10, T_0 = 4$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2 = 6T^2 + 1 \qquad (4)$$

Whose smallest positive integer solution is $(\widetilde{X}_0, \widetilde{T}_0) = (5,2)$ the general solution of (4) is given by

$$\widetilde{T}_n = \frac{1}{2\sqrt{6}} g_n, \widetilde{X}_n = \frac{1}{2} f_n$$

Where

$$f_n = (5+2\sqrt{6})^{n+1} + (5-2\sqrt{6})^{n+1}$$

$$g_n = (5+2\sqrt{6})^{n+1} - (5-2\sqrt{6})^{n+1},$$

$$n = -1, 0, 1 \ldots$$

Applying Brahmagupta lemma between $(\widetilde{x}_0, \widetilde{y}_0)$ and $(\widetilde{x}_n, \widetilde{y}_n)$ the other integer solutions of (3) are given by

$$x_{n+1} = 5f_n + \frac{12}{\sqrt{6}} g_n \qquad (5)$$

$$y_{n+1} = 2f_n + \frac{5}{\sqrt{6}} g_n \qquad (6)$$

From (2), (5) and (6) the values of x and y satisfying (1) are given by

$$x_{n+1} = 11f_n + \frac{27}{\sqrt{6}} g_n$$

$$y_{n+1} = 9f_n + \frac{22}{\sqrt{6}} g_n$$

The recurrence relations satisfied by x and y are given by

$$x_{n+3} - 10x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 10y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x_n and y_n satisfying (1) are given in the Table: 1 below.

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Integral Points on the Ternary Quadratic Diophantine Equation $y^2 = 33x^2 + 4^t$, $t \geq 0$

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***Abstract:** The binary quadratic equation $y^2 = 33x^2 + 4^t$ representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of Generalized Fibonacci sequences of numbers, Generalized Lucas sequences of numbers.*

***Keywords:** Binary quadratic integral solutions, generalized Fibonacci Sequences of numbers, generalized Lucas Sequences of numbers.*

AMS Mathematics Subject Classification: 11D09

Notations

$GF_n(k,s)$: Generalized Fibonacci Sequences of rank n.

$GL_n(k,s)$: Generalized Lucas Sequences of rank n.

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m))]}{6}$$

$$\Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

$$S_n = 6n(n-1) + 1$$

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1,2,4]. In [3] infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2 = 3x^2 + 1$. In [5] a, special Pythagorean triangle is obtained by employing the integral solutions of $y^2 = 182x^2 + 14$. In [6] different pattern of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of $y^2 = 14x^2 + 4$. In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 33x^2 + 4^t$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

II. METHODS OF ANALYSIS

Consider the binary quadratic equation

$$y^2 = 33x^2 + 4^t,\ t \geq 0 \qquad (1)$$

with least positive integer solutions is

$$x_0 = 4(2)^t, \qquad y_0 = 23(2)^t$$

To obtain the other solutions of (1).

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Observations on the Hyperbola, y2 = 14x2 + 16t, t >=0 $y^2 = 14x^2 + 16^t, t \geq 0$

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***Abstract:** The binary quadratic equation $y^2 = 14x^2 + 16^t$ representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of Generalized Fibonacci sequences of numbers, Generalized Lucas sequences of numbers.*
***Keywords:** Binary Quadratic Integral Solutions, Generalized Fibonacci Sequences of Numbers, Generalized Lucas Sequences of Numbers, Integral Solutions.*

MS Mathematics Subject Classification:11D09

Notations

$GF_n(k,s)$: Generalized Fibonacci Sequences of rank n.

$GL_n(k,s)$: Generalized Lucas Sequences of rank n.

$$t_{m,n} = n\left(I + \frac{(n-1)(m-2)}{2}\right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m))]}{6}$$

$$\Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

$$S_n = 6n(n-1) + 1$$

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D non-square positive integer has been studied by various mathematics for its non-trivial integer solutions.

When D takes different integral values $[1,2,4]$. In $[3]$ infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the $y^2 = 14x^2 + 1$. In [5] a special Pythagorean triangles is obtained by employing the integral solutions of $y^2 = 182x^2 + 14^t$.

In [6] different patterns of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of $y^2 = 14x^2 + 4$. In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 14x^2 + 16^t$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

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Observation on the Binary Quadratic Equation $y^2 = 105x^2 + 4^t, t \geq 0$

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***Abstract:** The binary quadratic equation is considered and a few interesting properties among the solutions are presented.*
***Keywords:** Binary quadratic, integral solutions, Generalized Fibonacci sequences, Generalized Lucas Sequences.*

I. INTRODUCTION

The Binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-5]. In this context one may also refer [5]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation representing a hyperbola. A few interesting properties among the solutions are presented.

II. NOTATIONS

$t_{m,n}$: Polygonal number of rank n with size m

P_n^m : Pyramidal number of rank n with size m

Pr_n : Pronic number of rank n

S_n : Star number of rank n

$Ct_{m,n}$: Centered Pyramidal number of rank n with size m

$GF_n(k,s)$: Generalized Fibonacci sequence number of rank n

$GL_n(k,s)$: Generalized Lucas sequence number of rank n

III.METHOD OF ANALYSIS

The binary non-homogeneous quadratic Diophantine equation represents a hyperbola to be solved for its non-zero integral solutions is

$$y^2 = 105x^2 + 4^t, t \geq 0 \tag{1}$$

The smallest positive integer solution (x_0, y_0) of (1) is

$$x_0 = 4(2^t), y_0 = 41(2^t) \tag{2}$$

To obtain the other solutions of (1), consider the pellian equation

$$y^2 = 105x^2 + 1 \tag{3}$$

Applying the Brahmagupta lemma between the solutions (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{4(2^t)}{2} f_n + \frac{41(2^t)}{2\sqrt{105}} g_n$$

$$y_{n+1} = \frac{41(2^t)}{2} f_n + \frac{420(2^t)}{2\sqrt{105}} g_n$$

THE BINARY QUADRATIC DIOPHANTINE EQUATION $y^2 = 272x^2 + 16$

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***stract** - The binary quadratic equation $=272x^2+16$ is considered and a few interesting ...rties among the solutions are presented. Employing ... integral solutions of the equation under ...siderations a few patterns of Pythagorean triangle are ...rved.*

...Words: Binary, Quadratic, Pyramidal numbers, integral ...tions

INTRODUCTION

...e binary quadratic equation of the form $y^2 = Dx^2 + 1$ ...ere D is non –square positive integer has been ...died by various mathematicians for its non-trivial ...egral solutions when D takes different integral values ...5]. In this context one may also refer [4,10]. These ...sults have motivated us to search for the integral ...utions of yet another binary quadratic equation $^2=272x^2+16$ representing a hyperbola .A few ...eresting properties among the solution are presented. ...ploying the integral solutions of the equation ...sideration a few patterns of Pythagorean triangles are ...tained.

1.1 Notations

- $t_{m,n}$: Polygonal number of rank n with size m
- P_n^m : Pyramidal number of rank n with size m
- Pr_n : Pronic number of rank n
- S_n : Star number of rank n
- $CP_{m,n}$: Centered Pyramidal number of rank n with size m
- $GF_n(k,s)$: Generalized Fibonacci sequence of rank n
- $GL_n(k,s)$: Generalized Lucas sequence of rank n

2. METHOD OF ANALYSIS

...onsider the binary quadratic Diophantine equation is

$$y^2 = 272x^2 + 16 \quad (1)$$

whose smallest positive integer solutions of (x_0, y_0) is,

$$x_0 = 8, y_0 = 132 \quad (2)$$

To obtain the other solutions of (1),Consider pellian equation is

$$y^2 = 272x^2 + 1 \quad (3)$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 2, \tilde{y}_0 = 33$$

whose general solution $(\tilde{x}_n, \tilde{y}_n)$ of (3) is given by,

$$\tilde{x}_n = \frac{1}{272} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (33 + 2\sqrt{272})^{n+1} + (33 - 272)^{n+1}$$

$$g_n = (33 + 2\sqrt{272})^{n+1} - (33 - 2\sqrt{272})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = 4f_n + \frac{66}{\sqrt{272}} g_n \quad (4)$$

$$y_{n+1} = 9f_n + \frac{40}{\sqrt{20}} g_n \quad (5)$$

Therefore (4) becomes

$$\sqrt{272} x_{n+1} = 4\sqrt{272} f_n + 66 g_n \quad (6)$$

Replace n by $n+1$ in (6),we get

$$\sqrt{272} x_{n+2} = 4\sqrt{272} f_{n+1} + 66 g_{n+1}$$

$$= 4\sqrt{272}(33 f_n + 2\sqrt{272} g_n) + 66(33 g_n + 2\sqrt{272} f_n)$$

$$\sqrt{272} x_{n+2} = 264\sqrt{272} f_n + 4354 g_n \quad (7)$$

Replace n by $n+1$ in (7),we get

INTEGRAL SOLUTIONS OF THE DIOPHANTINE EQUATION $Y^2=20x^2+4$

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Abstract: The binary quadratic equation $y^2 = 20x^2 + 4$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under considerations a few patterns of Pythagorean triangles are observed.

Keywords: Binary, Quadratic, Pyramidal numbers, integral solutions

INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values. In this context one may also refer [4, 10]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2=20x^2+4$ representing a hyperbola. A few interesting properties among the solution are presented. Employing the integral solutions of the equation consideration a few patterns of Pythagorean triangles are obtained.

Notations

- Polygonal number of rank n with size m
- Pyramidal number of rank n with size m
- Pronic number of rank n
- Star number of rank n
- Centered Pyramidal number of rank n with
-): Generalized Fibonacci sequence of rank n
-): Generalized Lucas sequence of rank n

METHOD OF ANALYSIS

Consider the binary quadratic Diophantine equation is

$$y^2 = 20x^2 + 4 \quad (1)$$

Whose smallest positive integer solutions of (x_0, y_0) is,

$$x_0 = 4,\ y_0 = 18 \quad (2)$$

To obtain the other solutions of (1), Consider pellian equation is

$$y^2 = 20x^2 + 1 \quad (3)$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 2,\ \tilde{y}_0 = 9$$

Whose general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{20}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (9+2\sqrt{20})^{n+1} + (9-2\sqrt{20})^{n+1}$$

$$g_n = (9+2\sqrt{20})^{n+1} - (9-2\sqrt{20})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by,

$$x_{n+1} = 2f_n + \frac{9}{\sqrt{20}} g_n \quad (4)$$

$$y_{n+1} = 9f_n + \frac{40}{\sqrt{20}} g_n \quad (5)$$

Therefore (3) becomes

$$\sqrt{20}x_{n+1} = 2\sqrt{20} f_n + 9g_n \quad (6)$$

Replace n by $n+1$ in (6), we get

$$\sqrt{20}x_{n+2} = 2\sqrt{20} f_{n+1} + 9g_{n+1}$$

$$= 2\sqrt{20}(9f_n + 2\sqrt{20}g_n) + 9(9g_n + 2\sqrt{20}f_n)$$

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OBSERVATION ON THE POSITIVE PELL EQUATION $y^2 = 35x^2 + 46$

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ABSTRACT

The binary quadratic equation represented by the positive pellian $y^2 = 35x^2 + 46$ is analyzed for its distinct integer solutions. A few interesting relation among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas

KEYWORDS: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solution.

1. INTRODUCTION

The binary quadratic Diophantine equations are rich is variety. The binary quadratic equation of the form $y^2 = Dx^2 + 1$.where D is non square positive integer has been satisfied by various mathematician for its non-trivial integral solution. When D takes different integral values[1-4].In [5-11] the binary quadratic non-homogeneous equation representing hyperbolas respectively are studied for their non-zero integral solutions. This communication concerns with yet another binary quadratic equation given by $y^2 = 35x^2 + 46$. The recurrence relation satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

2. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 46 \quad (1)$$

The smallest positive integer solutions of (1) are

$$x_0 = 1,\ y_0 = 9$$

To obtain the order solution of (1), consider the pellian equation

$$y^2 = 35x^2 + 1 \quad (2)$$

Whose initial solution is given by

$$\tilde{x}_0 = 1,\ \tilde{y}_0 = 6$$

The general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

Where

$$f_n = (6+\sqrt{35})^{n+1} + (6-\sqrt{35})^{n+1}$$
$$g_n = (6+\sqrt{35})^{n+1} - (6-\sqrt{35})^{n+1}, n = -1,0,1,\ldots\ldots,$$

bservations on the Non-homogeneous binary Quadratic Equation $8x^2-3y^2=20$

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ract - A Non-homogeneous binary quadratic equation represents hyperbola given by $8x^2-3y^2=20$ is analyzed for its non-istinct integer solutions. A few interesting relation between the solution of the given hyperbola, integer solutions for other s of hyperbola and parabola are obtained.

ords: **Non-homogeneous quadratic, binary quadratic, integer solutions.**

TRODUCTION

inary quadratic Diophantine equations of the form $ax^2-by^2=N,(a,b,N\neq 0)$ are rich in variety and have been analysed any mathematicians for their respective integer solutions for particular values of a , b and N. In this context, one may refer 3].

communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic ation given by $8x^2-3y^2=20$ representing hyperbola. A few interesting relations among its solutions are presented. wing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are sented.

ethod of Analysis

Diophantine equations representing the binary quadratic equation to be solved for its non-zero distinct integer solution is

$$8x^2-3y^2=20 \tag{1}$$

sider the linear transformations

$$x=X+3T\,,\ y=X+6T \tag{2}$$

m(1) and(2), we have

$$X^2=30T^2+19 \tag{3}$$

ose smallest positive integer solution is

$$X_0=7,T_0=1$$

obtain the other solutions of (3), consider the Pell equation

$$X^2=30T^2+1 \tag{4}$$

ose smallest positive integer solution is $\left(\widetilde{X}_0,\widetilde{T}_0\right)=(1,5)$

On the Positive Pell Equation $y^2 = 35x^2 + 29$

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Abstract - *The binary quadratic Diophantine equation represented by the positive Pellian $y^2 = 35x^2 + 29$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas.*

Key Words: **Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions**

1. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer, has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values [1-4]. For an extensive review of various problems, one may refer [5-17]. In this communication, yet another an interesting equation given by $y^2 = 35x^2 + 29$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

2. METHOD OF ANALYSIS

The Positive pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 29 \tag{1}$$

The smallest positive integer solutions of (1) are

$$x_0 = 1,\ y_0 = 8$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 35x^2 + 1 \tag{2}$$

whose initial solution is

$$\tilde{x}_0 = 1,\ \tilde{y}_0 = 6$$

The general Solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = (6+\sqrt{35})^{n+1} + (6-\sqrt{35})^{n+1}$$

$$g_n = (6+\sqrt{35})^{n+1} - (6-\sqrt{35})^{n+1},\ n = -1,0,1,\ldots$$

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PYTHAGOREAN TRIANGLE WITH 2A/P+H-LEG AS A NARCISSTIC NUMBER OF ORDERS 3, 4 AND 5

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ABSTRACT

This paper concerns with the problem of attaining Pythagorean triangle where, in each Pythagorean triangle expressions $\frac{2*Area}{Perimeter}+H-a$ Leg is represented by a Narcisstic numbers

Keywords: Pythagorean triangle, primitive and non primitive triangle, Narcisstic numbers.

I. INTRODUCTION

In number theory, Pythagorean triangles have been a very big interest to various mathematicians since it is a very big treasure house to hunt for. For various types of problem and ideas on Pythagorean triangle and special number, one may refer [1-12]. In this communication, we search for pairs of Pythagorean triangle so that in each pair $\frac{2*Area}{Perimeter}+Hypotenuse-a$ Leg is a Narcisstic number.

Definition: Narcisstic Number
An n digit number which is the sum of nth power of its digits is called an n- Narcisstic number. It is also known as Armstrong number.

II. METHOD OF ANALYSIS

Let T(x,y,z) be a Pythagorean triangle where

$$x=m^2-n^2, y=2mn, z=m^2+n^2 \qquad (1)$$

Denote the area, perimeter and hypotenuse of T(x,y,z) by A, P and H respectively.

$$\frac{2A}{P}+H-y=\alpha\text{, a Narcisstic number of orders 3,4 and 5.}$$

The problem under consideration is equivalent to solving the Diophantine equation

$$m(m-n)=\alpha \qquad (2)$$

Given α, it is possible to attain the values of m and n satisfying (2) knowing m, n and using the (1) obtains different Pythagorean triangle, each satisfying the relation $\frac{2A}{P}+H-y=\alpha$, a Narcisstic number. A few illustrations are presented in the Tables: 1, 2, 3 below:

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A Connection between Pythagorean Triangle and Sphenic Numbers

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Abstract: ***This paper concerns with the problem of obtaining many Pythagorean triangles where, in each Pythagorean triangles, the expression $\frac{2*\text{Area}}{\text{Perimeter}}+H-a$ Leg is represented by a Sphenic number and Sphenic palindrome number respectively. Also, we present the number of primitive and non-primitive Triangles.***

Keywords: Pythagorean triangles, Sphenic numbers, Sphenic Palindrome numbers, Primitive and non-primitive triangles.

I. INTRODUCTION

Number theory is the Queen of Mathematics. It is one of the largest and oldest branches of mathematics. We may note that there is one to one correspondence between the polygonal numbers and the sides of polygon. Apart from the above patterns of numbers, Polished numbers ,Nasty numbers and Dhuruva numbers have been considered in connections with Pythagorean triangles in [1-12]. In this communication, we search for patterns of Pythagorean triangles such that, in each of which, the expression $\frac{2*\text{Area}}{\text{Perimeter}}+H-a$ Leg is represented by a Sphenic number and Sphenic palindrome number and they are exhibited in sections A and B.

II. DEFINITION

1. *Palindrome Number:* Palindrome number is one that is the same when the digits are reversed.
2. *Sphenic Number:* A Sphenic number is a positive integer which is the product of exactly three distinct prime numbers.
3. *Sphenic Palindrome Number:* A Sphenic number which is palindrome is called a Sphenic palindrome number.

III.METHOD OF ANALYSIS

Let T(x,y,z) be a Pythagorean triangle where

$$x=m^2-n^2, y=2mn, z=m^2+n^2 \quad (1)$$

Denote the area, perimeter and hypotenuse of T(x,y,z) by A,P and H respectively.

Section A: $\frac{2A}{P}+H-y=\alpha$, a Sphenic number of orders 3 and 4.

The problem under consideration is mathematically equivalent to solving the Diophantine equation

$$m(m-n)=\alpha \quad (2)$$

Given α, it is possible to obtain the values of m and n satisfying (2). Knowing m, n and using (1) one obtains Pythagorean triangles, each satisfying the relation, $\frac{2A}{P}+H-y=\alpha$, a Sphenic number. It is worth to note that there are only four Pythagorean triangles as the Sphenic number is a product of exactly three distinct prime numbers, A few illustrations are presented in Table 1 below.

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On the Negative Pell Equation $y^2 = 48x^2 - 23$

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...act: The binary quadratic equation represents by negative Pellian $y^2 = 48x^2 - 23$ ***is analyzed for its distinct integer ...ions. A few interesting relations among the solution are given. Further, employing the solutions of the above hyperbola, we ...e obtained solutions of other choices of hyperbolas and parabolas.***

...ords: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.
...S Mathematics subject Classification (2010):11D09

I. INTRODUCTION

...e binary quadratic Diophantine equations (both homogeneous and non-homogeneous) are rich in variety. In [1-17] the binary ...dratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solutions. This ...mmunication concerns with yet another binary quadratic equation given by $y^2 = 48x^2 - 23$. The recurrence relations satisfied ...y the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

II. METHOD OF ANALYSIS

...e negative Pell equation representing hyperbola under consideration is

$$y^2 = 48x^2 - 23 \qquad (1)$$

...ose smallest positive integer solution is

$$x_0 = 1 \, , \, y_0 = 5$$

...o obtain the other solutions of (1), consider the Pell equation

$$y^2 = 48x^2 + 1$$

...ose smallest positive integer solution is

$$\widetilde{x}_0 = 1 \, , \, \widetilde{y}_0 = 7$$

...ose general solution is given by

$$\widetilde{x}_n = \frac{1}{2\sqrt{48}} g_n \, , \, \widetilde{y}_n = \frac{1}{2} f_n$$

...ere

$$f_n = \left(7 + \sqrt{48}\right)^{n+1} + \left(7 - \sqrt{48}\right)^{n+1}$$

$$g_n = \left(7 + \sqrt{48}\right)^{n+1} - \left(7 - \sqrt{48}\right)^{n+1}$$

...plying Brahmagupta lemma between (x_0, y_0) and $(\widetilde{x}_n, \widetilde{y}_n)$, the other integer solutions of (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{5}{8\sqrt{3}} g_n$$

$$y_{n+1} = \frac{5}{2} f_n + \frac{6}{\sqrt{3}} g_n$$

...e recurrence relations satisfied by x and y are given by

$$x_{n+3} - 14x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 14y_{n+2} + y_{n+1} = 0$$

...e numerical examples of x and y satisfying (1) are given in the Table: 1 below:

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ON BINARY DIOPHANTINE EQUATION

$8x^2 - 7y^2 = k^2 + 14k - 7$

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ABSTRACT

Non-homogeneous binary quadratic equation representing hyperbola given by $8x^2 - 7y^2 = k^2 + 14k - 7$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

Keywords: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

I. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a,b,c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $8x^2 - 7y^2 = k^2 + 14k - 7$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

II. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non- zero distinct integral solution is

$$8x^2 - 7y^2 = k^2 + 14k - 7 \tag{1}$$

Introduce the linear transformation

$$x = X + 7T, y = X + 8T \tag{2}$$

From (1) & (2) we have

$$X^2 = 56T^2 + k^2 + 14k - 7 \tag{3}$$

whose smallest positive integer solution is

$X_0 = k + 7, T_0 = 1$

To obtain the other solutions of (3), consider the pell equation,

$$X^2 = 56T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

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ON THE POSITIVE PELL EQUATION $y^2 = 34x^2 + 18$

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ABSTRACT:

The binary quadratic Diophantine equation represented by the positive pellian $y^2 = 34x^2 + 18$ is analyzed for its non-zero distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, the solutions of other choices of hyperbolas, parabolas and Pythagorean triangle are obtained.

KEYWORDS:

Binary quadratic, Hyperbola, Parabola, Integral solution, Pell equation.

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...N THE NEGATIVE PELL EQUATION $y^2 = 102x^2 - 18$

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...STRACT:

...binary quadratic Diophantine equation represented by the negative pellian $y^2 = 102x^2 - 18$ is analyzed for its ...zero distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions ...the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas and Pythagorean triangle.

...WORDS: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

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On Pairs of Rectangles and Armstrong Numbers

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stract

This paper concerns with the problem of obtaining s of rectangles, where, in each pair, the sum of the areas is esented by an Armstrong number with 3 and 4 digits pectively.

words: Pairs of rectangles, Armstrong number, Primitive angle, Non-Primitive rectangle.

Mathematics Subject Classification: 11D09

roduction:

Number is the essence of mathematical calculations. mbers have varieties of range and richness. Many numbers hibit fascinating properties, they form sequences, they form terns and so on [1-9]. A careful observer of patterns may e that there is a one to one correspondence between the mbers and the number of sides of the polygon. In particular, e may refer [10-15].

In this communication, we search for pairs of angles where, in each pair, the sum of the areas is resented by an Armstrong number with 3 and 4 digits pectively. The total number of primitive and non-primitive tangles is also given.

Definition: (Armstrong Number of Order 'n')

Let N be an n-digit number represented by

$$N = a_1 a_2 a_3 \ldots\ldots a_n$$

If $N = a_1^n + a_2^n + a_3^n \ldots + a_n^n$, then N is said to be an Armstrong number of order n.

In otherwords, A number that is the sum of its own digits each raised to the power of the number of digits.

Method of Analysis:

Let $R_1(x, y)$ and $R_2(X, Y)$ be two distinct rectangles whose corresponding areas are A_1, A_2.

Consider

$$A_1 + A_2 = \alpha \text{ (Armstrong Number)}$$

That is,

$$xy + XY = \alpha \quad (1)$$

Let q, r, s be three non-zero distinct positive integers and $r > s$.

Introduction of the linear transformations

$$x = s,\ y = 2q + s,\ X = r - s,\ Y = r + s \quad (2)$$

in (1) leads to

$$r^2 = \alpha - 2qs \quad (3)$$

Solving (3) for q, r, s and using (2), the corresponding values of rectangles R_1 and R_2 are obtained and presented in Table:1 below:

Table: 1 Rectangles

Armstrong number	R_1	R_2	Observations		Remarks
			Primitive	Non-Primitive	
153	(1, 33)	(10, 12)	R_1	R_2	Total number of Primitive rectangles =14 Total number of non-Primitive rectangles =16
	(8, 12)	(3, 19)	R_2	R_1	
	(2, 18)	(9, 13)	R_2	R_1	
	(1, 73)	(8, 10)	R_1	R_2	

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ON BINARY DIOPHANTINE EQUATION $7x^2 - 5y^2 = 8$

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ABSTRACT

Non-homogeneous binary quadratic equation representing hyperbola given by $7x^2 - 5y^2 = 8$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented.

Keywords: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

I. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $7x^2 - 5y^2 = 8$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

II. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$7x^2 - 5y^2 = 8 \tag{1}$$

Introduce the linear transformation

$$x = X + 5T, y = X + 7T \tag{2}$$

From (1) & (2) we have,

$$X^2 = 35T^2 + 4 \tag{3}$$

whose smallest positive integer solution is

$$X_0 = 12, T_0 = 2$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2 = 35T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

$$\tilde{X}_0 = 6, \tilde{T}_0 = 1$$

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ON BINARY QUADRATIC EQUATION $y^2 = 35x^2 + 29$

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ABSTRACT

The binary quadratic equation represented by the positive pellian $y^2 = 35x^2 + 29$ is analyzed for its distinct integer solutions. A few interesting relation among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas

KEYWORDS: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

1. INTRODUCTION

The binary quadratic Diophantine equations are rich in variety. The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non square positive integer has been satisfied by various mathematician for its non-trivial integral solution. When D takes different integral values [1-4]. In [5-15] the binary quadratic non-homogeneous equation representing hyperbolas respectively are studied for their non-zero integral solutions. This communication concerns with yet another binary quadratic equation given by $y^2 = 35x^2 + 29$.The recurrence relation satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

2. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 29 \tag{1}$$

whose smallest positive integer solution is

$$x_0 = 1,\ y_0 = 8$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 35x^2 + 1$$

whose smallest positive integer solution is

$$\tilde{x}_0 = 1,\ \tilde{y}_0 = 6$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(6+\sqrt{35}\right)^{n+1} + \left(6-\sqrt{35}\right)^{n+1}$$

On The Negative Pell Equation $y^2 = 30x^2 - 45$

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ABSTRACT:

The binary quadratic Diophantine equation represented by the negative pellian $y^2 = 30x^2 - 45$ *is analyzed for its non-zero distinct solutions. A few interesting relations among the solutions are given. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas, parabolas.*

KEYWORDS: *Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.*

I. INTRODUCTION

The binary quadratic equations of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been selected by various mathematicians for its non-trivial integer solutions when D takes different integral values[1-4].For an extensive review of various problems, one may refer[5-10]. In this communication, yet another interesting equation given by $y^2 = 30x^2 - 45$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are presented.

II. METHOD OF ANALYSIS

The negative Pell equation representing hyperbola under consideration is

$$y^2 = 30x^2 - 45 \tag{1}$$

whose smallest positive integer solution is

$$x_0 = 3 \,,\, y_0 = 15.$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 30x^2 + 1 \tag{2}$$

whose initial solution is given by

$$\widetilde{x}_n = 2 \,,\, \widetilde{y}_n = 11$$

The general solution $(\widetilde{x}_n, \widetilde{y}_n)$ of (2) is given by,

$$\widetilde{x}_n = \frac{1}{2\sqrt{30}} g_n \,,\, \widetilde{y}_n = \frac{1}{11} f_n$$

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ON THE NON HOMOGENEOUS BINARY QUADRATIC EQUATION

$4x^2-3y^2=37$

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Abstract:
This paper deals with the problem of obtaining non-zero distinct integer solutions to the non homogeneous binary quadratic equation represented by the Pell-like equation $4x^2-3y^2=37$. Different sets of integer solutions are presented. Employing the solutions of the above equation, integer solutions for other choices of hyperbolas and parabolas are obtained. A special Pythagorean triangle is also determined.

Keywords: Non-homogeneous, binary quadratic, Pell-like equation, hyperbola, parabola, integral solutions, Special numbers.

2010 Mathematics Subject Classification: 11D09

1. INTRODUCTION

The binary quadratic Diophantine equations (both homogeneous and non homogeneous) are rich in variety [1-6]. In [7-17] the binary quadratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solutions. These results have motivated us to search for infinitely many non-zero integral solutions of still another interesting binary quadratic equation given by $4x^2-3y^2=37$. The recurrence relations satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

2. METHOD OF ANALYSIS

Consider the non homogeneous binary quadratic equation

$$4x^2-3y^2=37 \tag{1}$$

Introducing the linear transformations

$$x=X\pm 3T,\ y=X\pm 4T \tag{2}$$

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SPECIAL CHARACTERIZATIONS OF POLYGONAL NUMBERS THROUGH PELL EQUATIONS

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Abstract:
In this paper, different choices of positive and negative Pell equations are considered. Employing the non-zero integer solutions of each of the above choices of positive and negative Pell equations, the relations among the special polygonal numbers are exhibited.

Keywords: Positive Pell equation, Negative Pell equation, Polygonal numbers, Integer solutions.

2010 Mathematics Subject Classification: 11D09

1. INTRODUCTION

Every researcher in Number Theory is familiar with the subject of Diophantine equations. In fact, Number theory is the great and rich intellectual heritage of man-kind and essentially a man-made world to meet his ideals of intellectual perfection. No doubt that number is the essence of mathematical calculations and one may discover beautiful patterns in numbers. Recognizing number patterns is also an important problem solving skill. It is worth to quote the remark "There is strength in numbers, but organizing those numbers is one of the great challenges" by the mathematician John C. Mather and one may call "Mathematics as the science of patterns" as remarked by Ronald Graham.

The numbers that can be represented by a regular geometric arrangement of equally spaced points are called Figurate numbers [1]. In [2], the relations among the pairs of special m-gonal numbers generated through the solutions of the binary quadratic equation $y^2 = 2x^2 - 1$ are determined. In [3], the relations among special figurate numbers through the equation $y^2 = 10x^2 + 1$ are obtained. In [4], employing the solutions of the Pythagorean equation, the relations between the pairs of special polygonal numbers such that the difference in each pair is a perfect square is obtained. Also, Bert Miller [5] has defined a number known as Nasty number as follows: A positive integer n is a Nasty number if $n = ab = cd$ and $a+b = c-d$ or $a-b = c+d$ where a, b, c and d are non-zero distinct positive integers.

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ON BINARY QUADRATIC EQUATION

$5x^2 - 6y^2 = 5$

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ABSTRACT

Non-homogeneous binary quadratic equation representing hyperbola given by $5x^2 - 6y^2 = 5$ is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented. Also, employing the solutions of the given equation, is constructed.

KEYWORDS: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation.

1. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a,b,c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $5x^2 - 6y^2 = 5$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, is constructed.

2. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non- zero distinct integral solution is

$$5x^2 - 6y^2 = 5 \tag{1}$$

Introduce the linear transformation

$$x = X + 6T, y = X + 5T \tag{2}$$

From (1) & (2) we have

$$X^2 = 30T^2 - 5 \tag{3}$$

whose smallest positive integer solution is

$$X_0 = 5, T_0 = 1$$

To obtain the other solutions of (3), consider the pell equation,

$$X^2 = 30T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

$$\tilde{X}_0 = 11, \tilde{T}_0 = 2$$

The general solution of (4) is given by,

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On Binary Diophantine Equation $3x^2 - 5y^2 = 12$

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Abstract: ***Non-homogeneous binary quadratic equation representing hyperbola given by*** $3x^2 - 5y^2 = 12$ ***is analyzed for its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbola and parabola are presented.***

Keywords: Binary quadratic, Hyperbola, Parabola, Integral solutions, Pell equation. AMS Mathematics subject Classification (2010):11D0

I. INTRODUCTION

The binary quadratic Diophantine equations of the form $ax^2 - by^2 = N, (a, b, c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. In this context, one may refer [1-18].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by, $3x^2 - 5y^2 = 12$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented.

II. METHOD OF ANALYSIS

The Diophantine equation representing the binary quadratic equation to be solved for its non-zero distinct integral solution is

$$3x^2 - 5y^2 = 12 \tag{1}$$

Introduce the linear transformation

$$x = X + 5T, y = X + 3T \tag{2}$$

From (1) & (2) we have,

$$X^2 = 15T^2 - 6 \tag{3}$$

whose smallest positive integer solution is

$$X_0 = 3, T_0 = 1$$

To obtain the other solutions of (3), consider the Pell equation

$$X^2 = 15T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

$$\widetilde{X}_0 = 4, \widetilde{T}_0 = 1$$

whose general solution is given by

$$\widetilde{T}_n = \frac{1}{2\sqrt{15}} g_n, \widetilde{X}_n = \frac{1}{2} f_n$$

where

$$f_n = \left(4 + \sqrt{15}\right)^{n+1} + \left(4 - \sqrt{15}\right)^{n+1}$$

$$g_n = \left(4 + \sqrt{15}\right)^{n+1} - \left(4 - \sqrt{15}\right)^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\widetilde{x}_n, \widetilde{y}_n)$, the other integer solutions of (1) are given by

$$\sqrt{15}x_{n+1} = 4\sqrt{15} f_n + 15 g_n$$

$$\sqrt{15}y_{n+1} = 3\sqrt{15} f_n + 12 g_n$$